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PRACTICAL MATHEMATICS

ISSUE No. 2

THEORY AND PRACTICE WITH MILITARY AND INDUSTRIAL APPLICATIONS

ADVANCED ARITHMETIC

Further Principles of Arithmetic

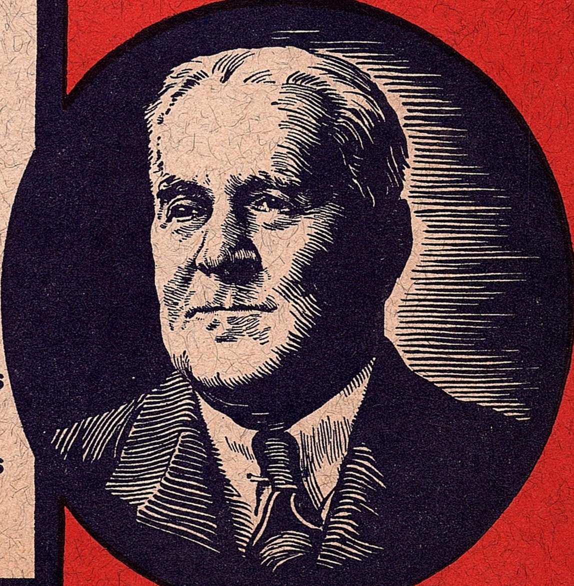
Decimals • Aliquots
Percentages
Averages • Ratios
Denominate Numbers

The Use of Logarithms
The Slide Rule

—ALSO—

Mathematical Tables and Formulas
Glossary of Mathematical Terms
Self-Tests and Arithmetic Problems

JAMES McGIFFERT, Ph.D.
Rensselaer Polytechnic Institute



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EDITOR: REGINALD STEVENS KIMBALL ED.D.

COURSE 2 **Practical Mathematics** VOLUME 1

REGINALD STEVENS KIMBALL, *Editor*

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CHATS WITH THE EDITOR

WEAKNESS in arithmetic computation and reasoning is cited as the chief reason for the failure of a "considerable percentage" of high-school graduates who have not met successfully tests for advancement in various branches of the military and naval services. "Send us men who know something about simple arithmetic and we can cut their training time" is the gist of statements issued from the training centers for aviation cadets.

Arithmetic, because it is a required subject in the elementary grades, is apt to be looked upon with scorn by grown men and women when they begin to think about adding to their mathematical knowledge. One prospective subscriber to this course assured me in all good faith that he didn't need any of the issues which preceded calculus, because he had "had" all the other subjects.

It is a common experience for all of us to discover that we have forgotten a great deal of what we learned years ago unless we have occasion to put it to use somewhere, somehow, some time. If you haven't been using arithmetic computation to any greater extent than balancing the family budget or the check-book, you've probably forgotten a great deal of the arithmetic you learned down in the grades. There's this consolation, however: a thing once learned can be relearned in much less time. That's why we've considered it safe to get along so rapidly in this review of arithmetic which we're presenting in the first two issues of PRACTICAL MATHEMATICS.

Before you begin to read the articles in this second issue, let me give you a word of caution. Make sure that you have really *mastered* all of the steps in the first issue. Once you know these, you'll find that it's much easier to go ahead with the more complicated processes that are discussed in the present issue.

This issue begins where the first issue left off. At the end of Dr. McGiffert's article on common fractions, you'll remember, we were just beginning to discover that fractions could be rather annoying at times if we didn't handle them properly. One way of handling them is to transform them into decimals. We begin, therefore, with a study of decimals in this issue. The handy little decimal point saves us a lot of bother when we have a great many fractions in a problem. The conversion table, showing the relationship between common fractions and decimals, in the back of this issue will help you further in your dealings with numbers.

Another matter which was introduced in the first issue is carried along in this present number. Mr. Farr's article on common weights and measures introduced a subject to which you probably haven't given much thought since your school-teacher used to require you to "learn the tables". Now, under the heading of "denominate numbers", you'll receive some guidance in changing these units into larger or smaller units as need arises.

Almost everyone working in war industries comes up against the necessity of using averages, ratio, and pro-

portion. The first article in this issue concludes with an introductory treatment of these subjects. In later issues, there will be additional consideration of these topics.

At this point, we're going to introduce you to two subjects which used to be postponed until late in the high-school course. Indeed, I've visited in many high schools in which the subjects aren't touched upon at all in the regular courses, either because the teacher considers them too difficult for his students or because he doesn't appreciate the advantages which the student will receive from being able to handle them.

Most people, when they hear logarithms mentioned, think of them as something highly mysterious which only a grey-bearded pedant could possibly understand. Actually, logarithms are anything but hard to handle. I've demonstrated time and again that fifth- and sixth-graders have little trouble in learning to use them. If you can add and subtract (and here's another reason for making sure of the "fundamental operations" treated in Issue Number One), you can put logarithms to use immediately.

Logarithms are time-savers. By the use of logarithms, you can perform complicated multiplications and divisions much more rapidly than by the ordinary methods. Don't pass over this article. Read it carefully, study it, practice on it, and then, in all further work in mathematics, make use of logarithms to save your overtaxed brain from unnecessary fatigue.

You will find that the five-place logarithmic table presented in Practical Mathematics will be sufficient for most of the reckoning which you will be called upon to do. A five-place table gives accuracy to four significant figures in the answer; we seldom have need for greater accuracy than this. If you are working upon a very technical problem, in which a higher degree of accuracy is demanded, however, you will have nothing new to learn. The use of a six-place, or even a ten-place table, is no different from the use of the five-place table given here. Practice with this table will give you the dexterity in locating mantissas which may be applied to a table of any size when you have occasion to need it.

Last fall, I had occasion to introduce a group of adult students in a "refresher course" to the slide rule. After a fifteen-minute demonstration, I turned them loose on their task of solving problems for the next day's class. "Is that all there is to the slide rule?" asked one amazed man. "Why, I had always thought one had to have a bunch of degrees after his name before he could dare to look at one." That wasn't quite all that there was to the slide rule, as later lessons demonstrated, but it was enough to get the class off to a good start.

Then we come to "The Measuring Rod". Issue Number One gave you, under this heading, a test on fundamental operations, to enable you to check up on yourself and determine the points on which you needed further practice, either because you were inaccurate or because you were too slow. The answers to all the exercises in Issue One are printed in this issue. (How well did you do?) Our present "Measuring Rod" presents a series of problems based on the whole field of arithmetic.

Let me remind you that we are not printing these "Measuring Rod" questions simply as a means of filling space. They are put here for a specific purpose: to give you a real opportunity to test your mastery of the articles you have been reading. The questions in each "Measuring Rod" have been selected with great care by a staff of experts. Each question has to meet the approval of every member of the staff, or out it

goes. In deciding what questions to include, we have been concerned to see that each problem is typical of the sort of applications of mathematical theory which you will be called upon to make in connection with your war-time job. These questions have been checked against the syllabi prepared by various federal departments and agencies and by a number of committees of teachers and mathematicians who have issued suggestions as to the type of mathematical training needed by prospective draftees and war workers.

Since you are taking this course in mathematics at home, without a teacher to stand over you and see that you complete the assignments, it will be necessary for you to be your own task-master. Make yourself do the required "home-work".

That would seem to be about enough for the present "assignment". For good measure, though, we're including another batch of tricky questions, with which you may entertain yourself and your friends. We hope that you're enjoying these. Some of them are "old as the hills"; others are especially prepared for these pages. If you haven't met them before, they're loads of fun. You can learn something from them, too. They'll teach you to be more careful in your use of numbers. Some of them prove that, while figures don't lie, liars do figure. Be on your guard when you're dealing with them, and don't be too much annoyed with yourself if you get "caught out" on some of them.

Now, for a look ahead. Having covered the important portions of arithmetic which you'll need to know in your industrial or military career, you are ready for some first steps in algebra. Issue Number Three, which will reach you ten days hence, introduces you to that subject.

Algebra isn't a foreign language; it's merely a way of broadening your use of arithmetic. We are selecting from the whole field of algebra the simple operations which you can put to immediate use on the job or in service.

Put thus simply, the subject of mathematics loses its foreboding appearance. It becomes your slave; not your master. Numbers can unlock doors for you, solving mysteries. In themselves, they are not mysterious.

The chief reason that most people have neglected mathematics in the past has been because they weren't helped to see just how useful mathematics was. Those people who boast or apologize because they "have no head for numbers" are usually amazed to find that they really can understand the subject when it's presented to them with applications to their own affairs.

A recent investigation of "likes and dislikes" of high-school students shows that mathematics is at one and the same time the most liked and the most detested subject in the school curriculum. It seems always to have been the case that people either enjoyed work in mathematics and wanted more of it or that they had no use for it at all.

For generations, school children have taken delight in chanting the little rhyme,

Multiplication is vexation;
 Subtraction is as bad;
The rule of three, it puzzles me;
 And fractions drive me mad.

With this attitude, they have hastened away from mathematics at the first opportunity and have proceeded blithely to forget all of the subject except those parts which they have had to use in their every-day affairs.

With the outbreak of the present war, the situation has changed. With point rationing and ceiling prices confronting us, everyone has to give more attention to numbers than he ever did before. The demands of the war industries, the complexities of private

business, the operations of the armed forces—all combine to give new emphasis to the importance of mathematics.

Through the pages of PRACTICAL MATHEMATICS, we hope to aid you in your new attack on the subject. We'll be glad to hear from you as to how successful we are in accomplishing our aim. There's one element which we can't supply, however. Learning comes through practice, and this practice you will have to do for yourself.

R.S.K.

ABOUT OUR AUTHORS

JAMES McGIFFERT, whose portrait appeared on the cover of this issue of PRACTICAL MATHEMATICS, has been associated with Rensselaer Polytechnic Institute ever since his graduation from that institution in 1891, serving successively as Instructor, Assistant Professor, Associate Professor, and Professor. Since 1930, he has been designated as Professor of Graduate Mathematics and Counselor and Adviser of the Mathematics Department.

Born in Stockport, N. Y., in 1868, he qualified for the degree of Civil Engineer at Rensselaer at the age of 23. He studied graduate mathematics at Johns Hopkins University, received the degrees of Bachelor of Arts and Master of Arts from Harvard University, and the degree of Doctor of Philosophy from Columbia University. He is a member of three honorary scholastic fraternities, Sigma Xi, Tau Beta Pi, and Theta Nu Epsilon, as well as of the American Mathematical Society and the Mathematical Association of America.

Dr. McGiffert is widely known as the author of *Plane and Solid Analytic Geometry* and of *Higher Algebra* as well as of several pamphlets on mathematics and a number of articles in mathematical journals. He was for a number of years a member of the editorial board of the *National Mathematics Magazine*. That his interests are not limited to the field of mathematics is attested by the fact that he is also the President of the Troy Society for Spoken English. He has lectured in many places throughout the country.

IN the early issues, PRACTICAL MATHEMATICS presents feature articles by Robert N. Farr. Farr was born in Iowa City, July 4, 1916, and as most of his career is still ahead of him, we cannot regale the reader with any formidable list of his mathematical achievements. Much of his talent for science and mathematics he inherits from his mother, Wanda K. Farr, famous for her work on the process by which living plants manufacture cellulose.

Farr's passage through the educational stages was almost meteoric. He graduated from the Peekskill Military Academy when he was fourteen years old, and Stanford University before he was twenty. At Stanford, he majored in mathematics, and by 1935 had earned the degrees of Bachelor of Arts and Master of Science. He is the author of such awesome papers as "Bernoulli's Theorem of Theoretical Probability", "Graphical Solutions Used as a Parallel of Analytical Solutions in a Study of Higher Mathematics", and "The Prismatoid and its Mensuration".

After Stanford, Farr became engaged in editorial labors for a number of publishing houses. In late 1942, he came to the National Educational Alliance to work with Dr. Kimball on PRACTICAL MATHEMATICS.

FURTHER PRINCIPLES OF ARITHMETIC

By James McGiffert, Ph.D.

DECIMAL FRACTIONS To convert an ordinary fraction to decimals, we simply divide the numerator by the denominator, annexing to the numerator as many zeros as may be necessary to permit of even division and expressing the quotient as a decimal. Thus, $\frac{1}{5}$ equals 1.0 divided by 5, or 0.2; $\frac{1}{4}$ equals 1.0 divided by 4, or 0.25; $\frac{1}{8}$ equals 1.000 divided by 8, or 0.125, etc. It is a good plan to memorize some of the more commonly used fractions so that it will not be necessary to stop in the midst of a computation to work out the answer to each problem as it appears. (Note table on page 125.) By the same rule, we may readily discern that $\frac{3}{5}$ equals 3.0 divided by 5, or 0.6, etc.

$$\begin{array}{r} 0.2 \\ 5\overline{)1.0} \end{array} \qquad \begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \end{array} \qquad \begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Conversions

To convert an ordinary fraction to a percentage, we remember that *per cent* comes from the Latin *per centum*, meaning "by the hundred", and thus realize that, if we annex two zeros, we may achieve our conversion in the same manner.

$$\begin{array}{l} .20 \text{ or } 20\% \\ 5\overline{)1.00} \end{array} \qquad \begin{array}{l} .25 \text{ or } 25\% \\ 4\overline{)1.00} \end{array} \qquad \begin{array}{l} .12\frac{1}{2} \text{ or } 12\frac{1}{2}\% \\ 8\overline{)1.00} \end{array}$$

To convert a decimal expression to a common fraction, we merely reverse the procedure, expressing the decimal as a numerator over a denominator which expresses in some multiple of ten the number indicated by the position of the decimal and then reduce the fraction to lowest terms, as:

$$0.125=\frac{125}{1000}=\frac{25}{200}=\frac{5}{40}=\frac{1}{8}$$

$$0.4628=\frac{4628}{10000}=\frac{2314}{5000}=\frac{1157}{2500}$$

Operations with decimals

When decimal fractions are used, the fundamental operations are performed substantially as with integers (see Issue Number One, pages 1 to 24). The few additional rules will be considered briefly in the following paragraphs.

ADDITION OF DECIMALS

Addition of decimals presents no new difficulties to one who has mastered the general principles of addition. There is only one new rule to keep in mind: *line up the decimal points one under another and proceed as in ordinary addition*. Thus, to add 152.763, 41.9842, 617.2, and 8.24, we have simply to arrange the figures in a column, get the sums of the digits in the various columns, carrying the left-hand digits in the case of numbers greater than 9, and set down the total with the decimal point directly under the points in the column. Any of the various forms of addition shown in Issue Number One on pages 3 to 10 may be employed.

```
152.763
 41.9842
617.2
  8.24
--------
       2
      7
    18
   20
  18
 10
7
--------
820.1872
```

TEST YOUR ABILITY TO ADD DECIMALS WITH THESE PROBLEMS

1 From what number must I subtract 5.1736 to leave 8.1964?
2 From what number must I subtract 6.231 to leave 9.6648?
3 From what number must I subtract 74.213 to leave 25.787?
4 A man pumps out of a cistern in one hour 243.75 gallons; in the next hour, 227.5 gallons; in 45 minutes more, 137.75 gallons; and the cistern is empty. How many gallons of water were in it?
5 The inside diameter of a steel tubing is 6.64″, the wall thickness is 0.034″. What is the outside diameter?

SUBTRACTION OF DECIMALS

In subtracting decimals, the same rule applies: line up the decimal points in the minuend and the subtrahend and proceed as in ordinary subtraction. If there are fewer "places" to the right of the decimal point in the minuend than in the subtrahend, annex zeros to fill the empty spaces. Thus, to subtract 41.9842 from 617.2, since there are four places to the right of the decimal point in the subtrahend and only one place to the right in the minuend, it is necessary to annex three 0's, making the 617.2 read 617.2000. When there are fewer places to the right of the decimal point in the subtrahend than in the minuend, the blank spaces may be considered as 0's. Since there is nothing to subtract, the numbers above the blank spaces are brought down unchanged.

$$\begin{array}{r} 617.2000 \\ -\ 41.9842 \\ \hline 575.2158 \end{array} \qquad\qquad \begin{array}{r} 41.9842 \\ -\ 8.24 \\ \hline 33.7442 \end{array}$$

TEST YOUR ABILITY TO SUBTRACT DECIMALS WITH THESE PROBLEMS

6 What must be subtracted from 1 to leave 0.5?
7 What must be subtracted from 1 to leave 0.53?
8 What must be subtracted from 1 to leave 0.532?
9 What must be subtracted from 1 to leave 0.5236?
10 What must be subtracted from 1 to leave 0.5235988?
11 Private Brown went into the Post Exchange and bought $9.75 worth of merchandise. He paid for it with his pay check, which totaled $41.85. How much change did he get?
12 The outside diameter of a steel tubing is 0.625″; the wall thickness is 0.042″. What is the inside diameter?

MULTIPLICATION OF DECIMALS

To multiply a number by a decimal, we point off (starting at the right) as many decimal places as are contained in the decimal. Thus, $1768 \times 0.1 = 176.8$, $1768 \times 0.01 = 17.68$, $1768 \times 0.001 = 1.768$, etc. Multi-

plication by decimals other than those with a significant digit of 1 would be performed similarly: as $1768 \times 0.2 = 353.6$, $1768 \times 0.03 = 53.04$, etc. If both numbers to be multiplied contain decimals, we proceed as in ordinary multiplication, but point off in the answer as many places as are pointed off in both multiplier and multiplicand. Since, in the case of 3.768 times 4.976, each has three decimal places, the result will have six decimal places.

$$\begin{array}{r} 3.768 \\ \times 4.976 \\ \hline 22\,608 \\ 263\,76 \\ 3391\,2 \\ 15072 \\ \hline 18.749\,568 \end{array}$$

It should be obvious that we should seldom need accuracy to one-millionth. We may drop the least important fractions here, following the rule that a digit less than 5 may be disregarded whereas digits greater than 5, when dropped, are compensated for by raising the last digit remaining by 1. Thus, successively dropping the unnecessary digits and making the compensations, we should have

$$\begin{array}{l} 18.749568 \\ 18.74957 \\ 18.7496 \\ 18.750 \\ 18.75 \end{array}$$

giving us 18.75 as the answer correct to two decimal places. In arriving at approximate answers, it is the rule to work the example to one more decimal place than we actually need. This is to help us determine whether the next figure would be greater or less than 5, so that we may know that the last figure we use is sufficiently accurate.

To take another example, 0.4132 times 0.187 calls to our attention another possible difficulty. Our result gives us only six significant figures, while we must have seven decimal places. Here we must insert a zero before the first figure in the result in order to obtain the proper number of places.

$$\begin{array}{r} 0.4132 \\ \times 0.187 \\ \hline 28924 \\ 33056 \\ 4132 \\ \hline 0.0772684 \end{array}$$

This answer, correct to two significant figures, would be arrived at by dropping the other digits, one at a time, as before. In that case, dropping the 4 would not change the value of the remaining final

figure; dropping the 8, however, would change the 6 to a 7; when the 7 is dropped, the 2 would change to a 3; dropping the 3 would not change the value of the 7; and we should have:

0.0772684
0.077268
0.07727
0.0773
0.077

With a little practice, one may discern at a glance whether or not the final figure is to be retained as it is or is to be increased by 1 without going through all of the intermediate steps.

Approximate multiplication—Since, in most multiplications involving decimals, we shall find it possible to drop several digits, it should be obvious that we do not need to carry out the entire multiplication in order to arrive at a fairly correct answer. Suppose we want to multiply 3.768 by 4.876, getting the answer correct to two decimal places. We might try dropping all of the decimals (compensating as before) and "guess" that our result would be slightly less than 4 times 5, or 20. This gives us no decimal places, however. Retaining one decimal place, with compensations, would give us 3.8 times 4.9, or 18.62. Our rule has suggested that we should carry our calculation to at least one more place than we desire. Let us, then, try 3.77 by 4.88, which would give us 18.3866, 18.387, or 18.39.

$$\begin{array}{r} 3.8 \\ \times 4.9 \\ \hline 342 \\ 152 \\ \hline 18.62 \end{array} \qquad\qquad \begin{array}{r} 3.77 \\ \times 4.88 \\ \hline 3006 \\ 3006 \\ 1508 \\ \hline 18.3866 \end{array}$$

Contracted multiplication—Another way of approximating the desired result in multiplying by decimals is known as "contracted multiplication". Since the figures to the left of the decimal point are the most important, we begin our multiplication with the left-hand digit of the multiplier, and multiply all of the digits in the multiplicand. Dropping the final figure of the multiplicand, we multiply the remaining digits by the second figure in the multiplier, and continue, successively dropping one more figure in the multiplicand each time. This approximation differs from the result we previously obtained by only $\frac{2}{100}$ (0.02), a relatively small amount. Performing the whole multiplication, we discover that these two approximations

are each off by $\frac{1}{100}$ (0.01), one in each direction from the actual result. In the examples below, the full multiplication is shown in the column at the left, while the column at the right shows the contracted form of multiplication, with each step clearly indicated so that you may follow the reasoning.

$$\begin{array}{r}
3.768 \\
\times 4.876 \\
\hline
22608 \\
26376 \\
30144 \\
15072 \\
\hline
18.372768 \\
18.37
\end{array}
\qquad\qquad
\begin{array}{rl}
3.768 & \\
4.876 & \\
\hline
15072 & (4\times 3.768) \\
3008 & (8\times 3.76) \\
259 & (7\times 3.7) \\
18 & (6\times 3) \\
\hline
18.357 & \\
18.36 &
\end{array}$$

TEST YOUR ABILITY TO MULTIPLY DECIMALS BY THESE EXERCISES

13 $0.6\times0.6=$?
14 $0.62\times0.36=$?
15 $0.3\times0.2=$?
16 $0.21\times0.4=$?
17 $2\times0.4=$?
18 $4\times0.5=$?
19 $3\times0.7=$?
20 $0.04\times2=$?
21 $1.6\times0.25=$?
22 $1.2\times0.875=$?
23 $7.5\times0.35=$?
24 $2.1\times0.037=$?
25 $0.2\times3=$?
26 $0.5\times6=$?
27 $0.7\times8=$?
28 $0.004\times15=$?
29 $1.4\times65=$?
30 $1.8\times57=$?
31 $0.4\times65=$?
32 $0.4\times2.35=$?
33 $0.9\times5.2=$?
34 $0.07\times1.2=$?
35 $3\times2.5=$?
36 $5\times3.5=$?
37 $2.4\times1.75=$?
38 $2.9\times2.35=$?

TEST YOUR KNOWLEDGE OF MULTIPLYING DECIMALS WITH THESE PROBLEMS

39 U. S. Navy specifications for phosphor bronze call for 85% copper, 7% tin, 0.06% iron, 0.2% lead, 0.3% phosphorus and the remainder to be zinc. How many pounds of each element are required to make 500 pounds of phosphor bronze?
40 On a certain flight, a Silverliner used 40.6 gallons of gasoline per hour. The time of the flight was 3 hr. 48 min. Find the amount of gasoline used.
41 Find the circumference of a 10″-pipe. Use $\pi=3.1416$.
42 The total wing span of a Curtiss 0–40 is 37.33 ft. The average width of the wing is 6.32 ft. Find the area of the wing.

DIVISION BY DECIMALS

When we were discussing division by fractions, we discovered that $5\div\frac{1}{2}$ equalled 10. Since 0.5 is another way of writing $\frac{1}{2}$, it should be obvious that $5\div0.5$ also equals 10.

$$0.5\overline{)\,5.0}\quad\text{quotient } 10.$$

To achieve this result when dividing by decimals, we have merely to remember this simple rule: *move the decimal point in the divisor a sufficient number of places to make the divisor a whole number. Move the decimal point in the dividend the same number of places, annexing 0's if necessary*. The reasoning is this:

$$18.7 \div 0.0875 \qquad \frac{18.7 \times 1000}{.0875 \times 1000} \qquad \frac{187000}{875}$$

```
             213.714
.0875)18.7000000
         17 50
          1 200
            875
            3250
            2625
             6250
             6125
              1250
               875
               3750
               3500
                250
```

In this instance, the 7-tenths indicates to us that our quotient is nearer to 214 than it is to 213. If we want accuracy in the decimal places, we may continue annexing 0's; however, the 1-one-hundredth indicates to us that 213.7 is a reasonably accurate answer. In most instances, it is a waste of effort to continue the division to several decimal places. Note that *the decimal point in the quotient appears directly above the decimal point in the dividend*. Let the degree of accuracy which is required in the problem on which you are working determine this matter for you in each instance.

TEST YOUR ABILITY TO DIVIDE DECIMALS BY THESE EXERCISES

43 3)0.3
44 6)2.4
45 3)3.6
46 7)0.21
47 250)1.25
48 6)0.9
49 8)6
50 4)7
51 4)2.5
52 2)41
53 6)0.09
54 75)3
55 24)1.5
56 0.6)0.42
57 0.2)0.8
58 0.3)0.66
59 0.4)1.2
60 0.7)7.28
61 8.1)4.86
62 2.1)6.3
63 1.2)3.6
64 0.3)0.027
65 16.2)1.134
66 0.06)0.3
67 0.5)8
68 3.5)7
69 0.25)1.5
70 2.25)4.5
71 0.8)0.4
72 0.5)0.9
73 0.4)2.2
74 3.5)1.02
75 2.5)7.2
76 0.4)0.0132
77 3.66)1.83
78 0.25)0.2
79 0.15)0.35
80 0.2)7
81 3.3)1
82 1.6)4
83 0.42)2.8
84 1.42)2.8
85 1.24)3.5
86 37.25)1
87 1.6)0.64
88 12.36)0.2472
89 2.72)0.64
90 4.44)0.44
91 1.47)0.7
92 13.25)0.4

TEST YOUR KNOWLEDGE OF DIVIDING DECIMALS WITH THESE PROBLEMS

93 A BT-14 plane carries 100 gallons of gasoline. If the fuel consumption is 28 gallons per hour, how long can the ship stay aloft? Carry to two decimal places.

94 One gallon of gasoline weighs 5.876 lbs. If the maximum load of a plane is 1250 lbs., how many gallons of gasoline can the plane carry? Carry to two decimal places.

95 A certain steel plate is 0.36″ thick. How many plates are there in a stack 6 feet high?

Percentages

Per centum, usually contracted to per cent, means "by the hundred" or "out of the hundred". One cent is the hundredth part of the dollar. A century is one hundred years (or, in slang, one hundred dollars). A centenarian is a man who has lived one hundred years. A centennial is the one hundredth anniversary of an event.

There are different ways of expressing per cent. Five per cent is often written as 5%, or as 0.05, or as $\frac{5}{100}$. It means "five out of every one hundred".

We express quantities as percentages when it is more convenient to do so, especially when we want to make comparisons between two quantities which are not easily commensurable. Sometimes we carry the idea to excess, as when we say $33\frac{1}{3}$ per cent when it would be much more simple and just as accurate to say $\frac{1}{3}$. As an example of the advantageous use of percentages, 5 per cent is obviously a greater proportion of a quantity than is $4\frac{3}{4}$ per cent, whereas it is not so readily obvious that 284 is a greater proportion of 5680 than 380 is of 8000.

TO FIND THE PER CENT OF A QUANTITY

Multiply the quantity by the expressed percentage—That is, given the problem to find 7 per cent of 5600, first express 7% as 0.07 and then multiply 5600 by it, pointing off two places to take care of the two decimal places in the multiplier, and thus finding the answer, 392. If our last two figures are zeros, we come out with a whole number; if they are numbers other than zero, we have a decimal fraction in our answer. In cases where it would be impossible to have a part of an object, it is necessary to express the answer as

the nearest whole number. Thus, in the case of 16% of 8642, we get the result, 1382.72. Expressed to the nearest whole number, this would be 1383. In the case of 6% of 1324, we should get 79.44, or, to the nearest unit, 79.

$$\begin{array}{r} 5600 \\ \times 0.07 \\ \hline 392.00 \end{array} \qquad \begin{array}{r} 8600 \\ \times 0.16 \\ \hline 518\ 52 \\ 864\ 2 \\ \hline 1382.72 \end{array} \qquad \begin{array}{r} 1324 \\ \times 0.06 \\ \hline 79.44 \end{array}$$

REVIEW YOUR KNOWLEDGE OF DECIMALS WITH THESE PROBLEMS

96 If you buy two packages of tobacco at seven cents each and a pipe for 65 cents how much change should you receive from a two-dollar bill?

97 A corporal in the Army takes out a government insurance policy for $5,000. $4.05 per month is deducted from his pay for the premium. How much will he have paid at the end of 35 months?

98 At a certain altitude and temperature, the true air speed is 14% greater than the calibrated speed. If the calibrated air speed is 172 mph, what is the true air speed?

99 $\frac{7''}{16}$ diameter holes are to reamed thirty-five ten-thousandths of an inch oversize. Find the finished diameter.

100 $\frac{15''}{64}$ diameter holes are to be reamed fifteen ten-thousandths of an inch oversize. Find the finished diameter.

101 At a replacement center, 0.42 of the soldiers were assigned to the infantry; 0.27 to the air force; 0.21 to a tank division and the remainder were assigned to a medical unit. The number of men sent to the medical unit was 480. How many men in all were moved from the replacement center?

102 If 7.48 gallons are equal to 1 cubic foot, and a cubic foot of water weighs 62.4 pounds, what is the weight of 1 quart of water?

TO FIND WHAT PER CENT A QUANTITY IS OF A LARGER QUANTITY

When we want to express a given quantity as a percentage of another quantity, we divide the given number by the number with which it is being compared. The decimal or mixed number resulting shows the fractional part. Removing the decimal point, we state the figure as a per cent. Thus, 3900 is found to be 0.65 or $\frac{65}{100}$ or 65% of 6000.

$$\begin{array}{r} 0.65 \\ 6000\overline{)3900.00} \\ 3600\ 0 \\ \hline 300\ 00 \\ 300\ 00 \\ \hline \end{array}$$

If a fraction results after the computation, carry the division one or two more places, expressing the entire decimal. Thus, $87\frac{1}{2}\%$ would be read as 0.875; $6\frac{1}{4}\%$ would be read as 0.0625, etc. We should say that 1800 is $37\frac{1}{2}\%$ or 0.375 of 4800. Similarly, 300 is $\frac{3}{16}$ or 0.1875 of 1600.

$$
\begin{array}{r}
0.375 \\
4800\overline{)1800.000} \\
\underline{1400\ 0} \\
360\ 00 \\
\underline{336\ 00} \\
24\ 000 \\
\underline{24\ 000}
\end{array}
\qquad\qquad
\begin{array}{r}
0.1875 \\
1600\overline{)300.0000} \\
\underline{160\ 0} \\
140\ 00 \\
\underline{128\ 00} \\
12\ 000 \\
\underline{11\ 200} \\
8000 \\
\underline{8000}
\end{array}
$$

Recurring decimals

Many fractions never come out to an even division in decimals. For example, $\frac{1}{3}$, no matter to how many places carried, still leaves a remainder, giving $0.3333333\ldots\ldots\frac{1}{3}$. When the same figure or the same group of figures in the same order are repeated over and over, the fraction is said to be a *recurring decimal.*

Aliquots

In many cases, percentages are most readily dealt with as fractions. Thus, $25\% = \frac{25}{100} = \frac{1}{4}$. To get 25% of a number, divide by 4. Any of the percentages shown in the table of aliquots (page 125) are handled more readily in this fashion. For a fuller discussion of this rule, refer to the table of short-cuts in division in Issue Number One of PRACTICAL MATHEMATICS (page 58). To show how much simpler this method is, let us find 25% of 316 by both methods.

$$
\begin{array}{r}
316 \\
\underline{\times 0.25} \\
15\ 80 \\
\underline{63\ 2} \\
79.00
\end{array}
\qquad\qquad
\begin{array}{r}
320 \\
\underline{\times 0.875} \\
1\ 600 \\
22\ 40 \\
\underline{256\ 0} \\
280.000
\end{array}
$$

$$\frac{1}{4}\times 316 = \begin{array}{r}79 \\ 4\overline{)316}\end{array} \qquad\qquad \frac{7}{\cancel{8}}\times \overset{40}{\cancel{320}} = 280$$

In the case of the complex al quots, after converting them to fractional form, divide by the denominator and multiply by the numerator.

TEST YOUR KNOWLEDGE OF PERCENTAGE WITH THESE PROBLEMS

103 A foot is 12 inches; a meter is 39.37 inches. A foot is what per cent of a meter?

104 If 18 airplanes out of a squadron of 27 airplanes are available for combat, what per cent of the squadron aircraft are available for combat?

105 A factory employs 875 men. Of these, 863 are present on one day. What percentage is this of the whole staff?

106 An error of not more than half a thousandth of an inch is allowed in measuring a length of about 2 inches. What is this error expressed as a per cent?

107 On a certain air raid, 15 planes were lost. This number was $7\frac{1}{2}\%$ of the number sent on the mission. How many planes were sent on the mission?

108 The indicated horsepower of an engine is 1,500. The actual effective horsepower is 16 per cent less than the indicated horsepower. Find the actual effective horsepower.

109 A machine shop employing 225 men is forced to employ 36 per cent more men. What is the increase in the number of employees?

110 23 per cent of a class of 1,900 cadets take their advanced training at Randolph Field. How many cadets does this represent?

111 A soldier on furlough buys a round-trip ticket for \$6.47 instead of two one-way tickets for \$3.78 each. What per cent is saved by buying the round-trip ticket?

112 A gasoline engine is found to be only 79% efficient. If it is rated at 120 horsepower, what actual horsepower does it develop?

113 The edges of a cube are increased in length by 10 per cent. What is the percentage increase in volume?

FINDING AVERAGES

To find the average of several items, we add them and divide by the number of items in the group. Thus, confronted with the problem of determining the average number of employees in six plants, we should add the separate figures for the six plants and then divide by 6. Since it is impossible to have $\frac{5}{6}$ of a worker, we should step our figure up to the next integer, since $\frac{5}{6}$ is greater than $\frac{1}{2}$. Had the fraction been less than $\frac{1}{2}$, we should simply have dropped the fraction.

$$\begin{array}{r} 342 \\ 876 \\ 239 \\ 516 \\ 723 \\ 417 \\ \hline 6\overline{)3113} \\ 518\frac{5}{6} = 519 \end{array}$$

Weighted average

Sometimes a simple average of the sort discussed in the previous paragraph does not give the best picture of a situation. Let us suppose that, in this same problem, plants 1 and 2 work a 40-hour week, plants 3 and 4 a 42-hour week, and plants 5 and 6 a 44-hour week. To get a true comparison of their productivity, expressed in man-hours, we should need to multiply the number of employees in each plant by the number of hours each was supposed to work. To find the average number of man-hours worked per week in each factory, we should divide by the number of factories, again 6. Here the fractional answer would have a meaning; hence, we let it stand. If we wished to determine the average number of hours each man worked in the given week, we should divide the total number of hours worked by the total number of employees, getting 41.99 or very close to 42 hours.

$$\begin{array}{r}
342\times 40=13680\\
876\times 40=35040\\
239\times 42=10038\\
516\times 42=21672\\
723\times 44=31834\\
417\times 44=\underline{18348}\\
6\overline{)130612}\\
21768\frac{4}{6}=\frac{2}{3}
\end{array}
\qquad\qquad
\begin{array}{r}
41.99\\
3113\overline{)130612.00}\\
\underline{12452}\\
6092\\
\underline{3113}\\
2979\ 0\\
\underline{2701\ 7}\\
277\ 30\\
\underline{270\ 17}\\
7\ 13
\end{array}$$

Perhaps it will be a little easier to follow the line of reasoning behind the use of the weighted average if you consider this situation. Knowing that a group of 13 men consumed 26 pounds of meat in a week, that a group of 8 men consumed 16 pounds, and that a group of 27 men consumed 54 pounds, we should still be nowhere in our reckoning if we divided the total of the three weights by three in order to determine how much meat to order for another group of 15 men, whereas adding the number of men and using that figure as a divisor for the total amount of meat would provide a dependable figure.

Short-cuts in finding averages

When several numbers for which we wish to find an average differ from each other but slightly, it is simpler to determine mentally the approximate average and then to deal with the deviations from this, adding the deviation to (or subtracting it from) the approximate average. Thus, in the case of a daily sales record such as shown in the example on page 77, we may see at a glance that the sales range is around $200.00 a day. Indicating in parallel columns the variations above or below that figure, we subtract the total of deviations below that amount ($29.18) from the total of deviations above that amount ($63.74), take the average deviation ($5.76), and add it to the original $200.00, getting $205.76.

Computation

	VARIATIONS	
	Above	*Below*
$211.89	11.89	
187.42		12.58
191.16		9.84
227.06	27.06	
214.79	14.79	
193.24		6.76
	63.74	29.18

Sales average about $200.00

$$\begin{array}{r} 63.74 \\ -29.18 \\ \hline 6\overline{)34.56} \\ \hline 5.76 \end{array}$$

$$\begin{array}{r} \$200.00 \\ -\quad 5.76 \\ \hline \$205.76 \end{array}$$

TEST YOUR KNOWLEDGE OF AVERAGES WITH THESE PROBLEMS

114 Find the average output per hour if eight machines in a shop turn out the following units per day: 375, 253, 425, 378, 368, 496, 319, 297.

115 An army officer making a tour of inspection of training stations travels 356 miles on Monday, 532 miles on Tuesday, 375 miles on Wednesday, 1,874 miles on Thursday, and 156 miles on Friday. What was his average distance per day?

116 The noon temperatures for successive days one week in New York City were 35°, 44°, 48°, 52°, 49°, 38°, 41°. What was the average daily temperature?

117 The weights of four men, composing the crew of a bomber, are respectively 166 lbs., 182 lbs., 139 lbs., and 177 lbs. What is the average weight of the crew?

118 In a small war production plant twelve employees are paid $65.94 a week each, twenty-one employees are paid $50.12 a week each, four employees are paid $112.87 a week each, and one employee is paid $157.21 a week. What is the average weekly wage in this plant?

119 Find the average of the following measurements of a length: 4.765 cm., 4.768 cm., 4.763 cm., 4.764 cm.

DENOMINATE NUMBERS

When a quantity is expressed in two or more values (as a measure of length in feet and inches or a measure of capacity in gallons, quarts, and pints), we have our choice of several ways of treating them. We may combine each unit as it stands, then converting the figure for any unit which could be expressed in terms of a larger unit; we may treat the smaller units as fractional parts of the larger ones; or we may convert the larger units into a greater number of the smaller.

In order to do any one of these, we must know the relationship

among the units (such as 12 inches in 1 foot, 60 minutes in 1 hour, etc.), as shown in the tables on page 62 of Issue Number One. Probably you are familiar with some of these relationships through constant usage; if you are like most people, however, many of those which you once learned in school have passed into the limbo of forgotten lore. Refresh your memory of these before you attempt to read and use the rest of this article.

Addition

If we have occasion to combine two lengths, such as 12 feet, 8 inches, and 9 feet, 6 inches, we may proceed by any of these methods:

a We may add together the feet, getting 21, and the inches, getting 14. Knowing that 14 inches are more than 1 foot, since 1 foot contains 12 inches, we change the 14 inches to 1 foot, 2 inches, adding this expression to the 21 feet which we already have, getting as our final result 22 feet, 2 inches.

$$\begin{array}{rr} & 12\text{ feet, } 8\text{ inches} \\ + & 9\text{ feet, } 6\text{ inches} \\ \hline & 21\text{ feet, } 14\text{ inches} \end{array}
\qquad
12\text{ inches}\overline{)14\text{ inches}}^{\;1\text{ foot, } 2\text{ inches}}$$

$$\begin{array}{rl} & 21\text{ feet} \\ + & 1\text{ foot, } 2\text{ inches} \\ \hline & 22\text{ feet, } 2\text{ inches} \end{array}$$

b We may treat the inches as fractional parts of a foot, expressing them either as common fractions or as decimal fractions. In our example, using common fractions, we come out with an improper fraction, which is treated like any improper fraction (see page 26) in arriving at the final answer.

$$\begin{array}{rl} 12\text{ feet, } 8\text{ inches} = & 12\frac{8}{12}\text{ feet} \\ +\ 9\text{ feet, } 6\text{ inches} = & 9\frac{6}{12}\text{ feet} \\ \hline & 21\frac{14}{12}\text{ feet} = 22\frac{2}{12}\text{ feet} = 22\frac{1}{6}\text{ feet} \end{array}$$

$$\begin{array}{rl} 12\text{ feet, } 8\text{ inches} = & 12.667\text{ feet} \\ +\ 9\text{ feet, } 6\text{ inches} = & 9.5\quad\text{feet} \\ \hline & 22.167\text{ feet} \end{array}$$

c We may reduce the whole expression to inches. Multiply the number of feet in each case by 12; then at the end divide the whole number of inches by 12 to determine the number of feet, treating the remainder as inches.

$$\begin{array}{rl} 12\text{ feet, } 8\text{ inches} = & 152\text{ inches} \\ +\ 9\text{ feet, } 6\text{ inches} = & 114\text{ inches} \\ \hline & 266\text{ inches} \end{array}
\qquad
12\text{ inches}\overline{)266\text{ inches}}^{\;22\text{ feet, } 2\text{ inches}}$$

In most instances, we are free to use whichever of these three methods is the easiest to use under given conditions.

TEST YOUR ABILITY TO ADD DENOMINATE NUMBERS BY THESE EXERCISES

120 Add: 4 gallons, 3 pints; 7 gallons, 3 quarts, $\frac{1}{2}$ pint; 9 gallons, $6\frac{1}{4}$ quarts, 1 pint; 5 gallons, 2 quarts, $4\frac{1}{4}$ pints.

121 Add: 4 hours, 43 minutes, 38 seconds; 5 hours, 14 minutes, 10 seconds; 16 hours, 24 minutes; 8 hours, 3 minutes, 14 seconds.

122 Add: 65°48′10″, 66°12′55″, 120°21′02″, 3°15′25″.

123 Add: 5 gross, 2 dozen; 8 gross, $9\frac{1}{2}$ dozen; 3 gross, $5\frac{1}{4}$ dozen; 11 gross, $\frac{1}{4}$ dozen.

Subtraction

In subtracting one item from another, the complication may arise that there are more of the smaller units in the subtrahend than in the minuend. In that case, we must "borrow" one or more of the larger units and convert it into the smaller units before we can perform the subtraction.

To subtract 19 days, 10 hours, from 24 days, 5 hours, we may go about the task as follows:

a From the 24 days we subtract 1 whole day, leaving 23 days, and convert the 1 day into 24 hours, which we add to the 5 hours which appeared in the original problem, getting 29 hours. Then our subtraction follows a simple procedure, taking hours from hours and days from days.

$$\begin{array}{rr} 24 \text{ days, } 5 \text{ hours} = & 23 \text{ days, } 29 \text{ hours} \\ -19 \text{ days, } 10 \text{ hours} = & -\underline{19 \text{ days, } 10 \text{ hours}} \\ & 4 \text{ days, } 19 \text{ hours} \end{array}$$

b Treating the hours as fractional parts of a day, we proceed as in any example involving common fractions.

$$\begin{array}{rrr} 24 \text{ days, } 5 \text{ hours} = & 24\frac{5}{24} \text{ days} = & 23\frac{29}{24} \text{ days} \\ -19 \text{ days, } 10 \text{ hours} = & -19\frac{10}{24} \text{ days} = & -19\frac{10}{24} \text{ days} \\ \hline & & 4\frac{19}{24} \text{ days} \end{array}$$

$$\begin{array}{rr} 24 \text{ days, } 5 \text{ hours} = & 24.2083 \text{ days} \\ -19 \text{ days, } 10 \text{ hours} = & -\underline{19.4175 \text{ days}} \\ & 4.7908 \text{ days} \end{array}$$

c The most laborious process in this case would be to convert the days into hours and then reconvert the answer into hours and days. This process is performed as follows:

$$\begin{array}{rcrcr} 24\text{ days, } 5\text{ hours} = & & 576\text{ hours} + 5\text{ hours} = & & 581\text{ hours} \\ -19\text{ days, } 10\text{ hours} = & & -456\text{ hours} + 10\text{ hours} = & & -466\text{ hours} \\ \hline & & & & 115\text{ hours} = 4\text{ days, } 19\text{ hours} \end{array}$$

TEST YOUR ABILITY TO SUBTRACT DENOMINATE NUMBERS BY THESE EXERCISES

124 From 360°, take 80°55′30″.
125 Subtract 3 gallons, 2 quarts, 1 pint from 10 gallons.
126 From 14 yards, 2 feet, $3\frac{1}{2}$ inches, take 3 yards, 1 foot, $2\frac{1}{4}$ inches.
127 Subtract 4 tons 1640 pounds, from 11 tons 200 lbs.

Multiplication

When we have occasion to multiply a quantity expressed in terms of various capacities, we frequently find that we have increased each capacity to the point that it is better to express it, so far as possible, in terms of the next larger unit.

If we have 7 containers each holding 7 gallons, 3 quarts, and 1 pint, we may wish to compute their combined capacity. We may multiply each of the three measures separately, then converting each in terms of the next larger unit, or we may again treat the whole thing in terms of the largest unit, expressing the other units as fractional parts, or we may convert all units to the smallest unit.

a Multiplying 7 gallons by 7 gives us 49 gallons; multiplying 3 quarts by 7 gives us 21 quarts or 5 gallons, 1 quart; multiplying 1 pint by 7 gives us 7 pints, or 3 quarts, 1 pint. Adding the three results gives us 54 gallons, 4 quarts, 1 pint, but, since 4 quarts equal 1 gallon, our final answer is 55 gallons, 1 pint.

$$\begin{array}{r} 7\text{ gallons} \\ \times 7 \\ \hline 49\text{ gallons} \end{array} \qquad 49\text{ gallons}$$

$$\begin{array}{r} 3\text{ quarts} \\ \times 7 \\ \hline 21\text{ quarts} \end{array} \qquad 4\overline{)21\text{ quarts}}^{\;5\text{ gallons, 1 quart}} \qquad 5\text{ gallons, 1 quart}$$

$$\begin{array}{r} 1\text{ pint} \\ \times 7 \\ \hline 7\text{ pints} \end{array} \qquad 2\overline{)7\text{ pints}}^{\;3\text{ quarts, 1 pint}} \qquad \begin{array}{r} 3\text{ quarts, 1 pint} \\ \hline 54\text{ gallons, 4 quarts, 1 pint} \end{array}$$

$$4\overline{)4\text{ quarts}}^{\;1\text{ gallon}} \qquad 55\text{ gallons, 1 pint}$$

b Treating 3 quarts as $\frac{3}{4}$ gallon is simple. Since 1 pint equals $\frac{1}{2}$ quart and 1 quart equals $\frac{1}{4}$ gallon, it follows that 1 pint equals $\frac{1}{2}\times\frac{1}{4}$ or $\frac{1}{8}$ gallon. Adding the fractions, we get $\frac{3}{4}+\frac{1}{8}=\frac{6}{8}+\frac{1}{8}=\frac{7}{8}$ gallon to be added to the 7 gallons. After multiplying, we have $\frac{49}{8}$ gallons, which converts to $6\frac{1}{8}$ gallons, which, added to the 49 gallons, gives $55\frac{1}{8}$ gallons.

$$\begin{array}{lllll}
7 \text{ gallons} & = 7 \text{ gallons} & = & 7 \text{ gallons} \\
3 \text{ quarts} & = \frac{3}{4} \text{ gallon} & = & \frac{6}{8} \text{ gallon} \\
1 \text{ pint} & = \frac{1}{2} \text{ quart} & = & \frac{1}{8} \text{ gallon} \\
\hline
 & & & 7\frac{7}{8} \text{ gallons}
\end{array}$$

$$\begin{array}{l}
7\frac{7}{8} \text{ gallons} \\
\times 7 \\
\hline
49\frac{49}{8} \text{ gallons} = 55\frac{1}{8} \text{ gallons}
\end{array}$$

Decimally, this is somewhat easier:

$$\begin{array}{llll}
7 \text{ gallons} & = 7 \quad \text{gallons} & = 7 & \text{gallons} \\
3 \text{ quarts} & = 0.75 \text{ gallon} & = 0.75 & \text{gallon} \\
1 \text{ pint} & = 0.5 \quad \text{quart} & = 0.125 & \text{gallon} \\
\hline
 & & 7.875 & \text{gallons}
\end{array}$$

$$\begin{array}{r}
7.875 \text{ gallons} \\
\times \quad 7 \qquad\qquad \\
\hline
55.125 \text{ gallons}
\end{array}$$

c Converting the gallons to quarts, then adding this amount to the number of quarts, and finally converting the whole amount to pints might prove feasible under some circumstances. By dividing the number of pints thus obtained by the number of pints in a gallon (8), we should achieve our answer directly.

$$7 \text{ gallons} = 7\times4 \text{ quarts} = 28 \text{ quarts}$$

$$\begin{array}{r}
28 \text{ quarts} \\
+\ 3 \text{ quarts} \\
\hline
31 \text{ quarts}
\end{array}$$

$$31 \text{ quarts} = 31\times2 \text{ pints} = 62 \text{ pints}$$

$$\begin{array}{r}
62 \text{ pints} \\
+\ 1 \text{ pint} \\
\hline
63 \text{ pint}
\end{array}$$

$$\begin{array}{r}
63 \text{ pints} \\
\times\ 7 \qquad \\
\hline
441 \text{ pints}
\end{array}
\qquad
2\overline{)441 \text{ pints}} = 220 \text{ quarts, 1 pint}
\qquad
4\overline{)220 \text{ quarts}} = 55 \text{ gallons}$$

or

$$8\overline{)441 \text{ pints}} = 55 \text{ gallons, 1 pint}$$

TEST YOUR ABILITY TO MULTIPLY DENOMINATE NUMBERS BY THESE EXERCISES

128 Multiply 8 yards, 5 feet, 2 inches by 7.

129 How many yards of cloth will be required to make one gross of Army-Navy "E" banners 2 yards, $2\frac{1}{2}$ feet, 5 inches in length?

130 Multiply 34°8′14″ by 9.

131 It is necessary to construct one template $6\frac{1}{4}''$ by 1′2″, and a second template three times as large. Find the size of the second template.

Division

In dividing a given quantity into parts, we may similarly face the problem of converting units.

If we have 5 bushels, 3 pecks, and 4 quarts to be divided into four equal lots, we have no trouble with 4 of the bushels. We may convert the odd bushel into pecks, then having 7 pecks, of which 4 could be distributed. This would leave us 3 pecks to be converted into quarts, after which our distribution may be easily completed, giving 1 bushel, 1 peck, 7 quarts for each of the four.

a

$$\begin{array}{r} 5 \text{ bushels} \\ -4 \text{ bushels} \\ \hline 1 \text{ bushel} \end{array} \qquad 1 \text{ bushel} = 4 \text{ pecks} \qquad 4\overline{)4 \text{ bushels}}^{\;1 \text{ bushel}}$$

$$\begin{array}{r} 3 \text{ pecks} \\ +4 \text{ pecks} \\ \hline 7 \text{ pecks} \\ -4 \text{ pecks} \\ \hline 3 \text{ pecks} \end{array} \qquad 3 \text{ pecks} = 3 \times 8 \text{ quarts} = 24 \text{ quarts} \qquad 4\overline{)4 \text{ pecks}}^{\;1 \text{ peck}}$$

$$\begin{array}{r} 24 \text{ quarts} \\ +\ 4 \text{ quarts} \\ \hline 28 \text{ quarts} \end{array} \qquad\qquad 4\overline{)28 \text{ quarts}}^{\;7 \text{ quarts}}$$

b Treating 3 pecks as $\frac{3}{4}$ bushel and 4 quarts as $\frac{1}{2}$ peck or $\frac{1}{2}\times\frac{1}{4}$ bushel, which equals $\frac{1}{8}$ bushel, we have

$$\begin{array}{lll} 5 \text{ bushels} & = 5 \text{ bushels} & = 5 \text{ bushels} \\ 3 \text{ pecks} & = \frac{3}{4} \text{ bushel} & = \frac{6}{8} \text{ bushel} \\ 4 \text{ quarts} & = \frac{1}{2} \text{ peck} & = \frac{1}{8} \text{ bushel} \\ \hline & & 5\frac{7}{8} \text{ bushels} = \frac{47}{8} \text{ bushels} \end{array}$$

$$\frac{47}{8} \text{ bushels} \div 4 = \frac{47}{8} \times \frac{1}{4} = \frac{47}{32} = 1\frac{15}{32} \text{ bushels}$$

The decimal computation is somewhat easier here:

5 bushels = 5 bushels	= 5	bushels
3 pecks = 0.75 bushel	= 0.75	bushel
4 quarts = 0.5 peck	= 0.125	bushel
	5.875	bushels

$$4\overline{)5.875 \text{ bushels}} = 1.46875 \text{ bushels}$$

c Converting the bushels to pecks, adding this amount to the number of pecks, and then converting the whole amount to quarts would be another way of attacking the problem. Then, dividing the whole amount by 4, and reconverting through the various units, we should have:

5 bushels = 5×4 pecks = 20 pecks

$$\begin{array}{r} 20 \text{ pecks} \\ +\ \underline{3 \text{ pecks}} \\ 23 \text{ pecks} \end{array}$$

23 pecks = 23×8 quarts = 184 quarts

$$\begin{array}{r} 184 \text{ quarts} \\ +\ \underline{4 \text{ quarts}} \\ 188 \text{ quarts} \end{array} \qquad 4\overline{)188 \text{ quarts}} = 47 \text{ quarts} = 5 \text{ pecks, } 1 \text{ quart}$$

$$4\overline{)5 \text{ pecks}} = 1 \text{ bushel, } 1 \text{ peck}$$

TEST YOUR ABILITY TO DIVIDE DENOMINATE NUMBERS BY THESE EXERCISES

132 Divide 12 gross $9\frac{3}{4}$ dozen by 3.

133 Divide 8 tons 720 pounds by 18.

134 Divide 11 bushels, 2 pecks, 1 quart by 2.

135 A template 30′9″ is to be marked off into twelve equal sections. How wide is each section?

RATIO AND PROPORTION

When we are confronted by the problem of comparing two or more quantities, we frequently find it helpful to make use of ratio and proportion. *Ratio* considers the relative sizes of two numbers. It is found by dividing one number by the number with which it is being compared. While ratio is sometimes written by placing the sign, :, between the two numbers under consideration, it may also be written (and to much greater advantage) in fractional form, and then may be treated like any fraction (see Issue Number One, page 27). The ratio of 6 to 12

$\left(\text{written } 6{:}12 \text{ or } \frac{6}{12}\right)$ is $\frac{1}{2}$; the ratio of 12 to 6 $\left(\text{written } 12{:}6 \text{ or } \frac{12}{6}\right)$ is 2. In both instances, we find that numerator and denominator have a common factor of 6, by which we then divide both.

Proportion is a statement that two ratios are equal. The most common occurrence of proportion is to be found in any example in which we reduce fractions to lowest terms or in which we find a common denominator and raise various fractions to equivalent fractions having that common denominator (again see Issue Number One, page 33).

Expressing proportions

If we wish to say that 8 is to 12 as 2 is to 3, we may write our ratio in either of the following forms:

$$8{:}12{::}2{:}3$$

$$\frac{8}{12}=\frac{2}{3}$$

By adopting the fractional form, we save ourselves the difficulty of learning a complete new set of rules for solving proportions. We may clear the equation of fractions by multiplying both numerator and denominator of each fraction by the number appearing in the denominator of the other. Thus,

$$\frac{8\times3}{12\times3}=\frac{2\times12}{3\times12}$$

The denominators thus being the same, each side of the equation may be multiplied by this amount to "clear" of denominators.

$$\frac{(12\times3)8\times3}{12\times3}=\frac{2\times12(12\times3)}{3\times12}$$

Then we have the simple statement

$$8\times3=2\times12$$

Since we may always clear of denominators in this fashion, we simplify our procedure by multiplying the numerator of each fraction by the denominator of the other, taking the other steps for granted.

In solving a problem in which one of the members of a proportion is unknown, we may place the three known items in our fraction, indicating the unknown item by a question mark (?), a letter, or other symbol. Following the line of reasoning in the previous paragraph, we then clear of fractions and solve for the unknown quantity.

Simple proportion

If we wish to determine the cost of 28 bags of flour when we already know that 16 bags cost \$17.92, we might make use of our knowledge of elementary arithmetic and divide the cost of the 16 bags by 16; then multiply the result by 28.

$$\begin{array}{r} \$\,1.12 \\ 16\overline{)\$17.92} \\ 16 \\ \hline 1\,9 \\ 1\,6 \\ \hline 32 \\ 32 \\ \hline \end{array} \qquad\qquad \begin{array}{r} \$1.12 \\ \times\ 28 \\ \hline 8\,96 \\ 22\,4 \\ \hline \$31.36 \end{array}$$

This is a rather lengthy and complicated process, which we may simplify by expressing the statement in an equation in the form of a proportion. Thus expressed, our problem would appear in this form:

$$\frac{16}{28}=\frac{\$17.92}{?}$$

Here it is easier to reduce the first fraction before proceeding, thus simplifying the computations necessary to produce the final solution. Since both 16 and 28 are divisible by 4, we get

$$\frac{16}{28}=\frac{4}{7}$$

Substituting this in the proportion, we have

$$\frac{4}{7}=\frac{\$17.92}{?}$$

Ridding our equation of denominators, we have

$$\begin{gathered} 4\times(?)=7\times\$17.92 \\ 4?=\$125.44 \end{gathered}$$

Dividing by 4, we find that the missing quantity is \$31.36, so that our proportion reads

$$\frac{16}{28}=\frac{\$17.92}{\$31.36}$$

Inverse proportion

Sometimes quantities vary inversely; that is, as one quantity increases, the other decreases. In the case of a piece of work to be done, for example, we know that, in general, the greater the number of men employed on the job the less time will be consumed in carrying the job to completion. So long as we compare the number of men in the first instance to the number of men in the second instance and the number of days in the first instance to the number of days in the second instance, we cannot go astray in our calculations. Taking a specific instance: if 18 men can complete a job in 12 days, how many men must be employed to do the job in 6 days? Seeing instantly that there are 18×12 man-days involved, we place these two figures in our proportion in such a way that they must be multiplied together when we simplify the equation by eliminating the denominators, thus:

$$\frac{18}{\cdot}=\frac{\cdot}{12} \quad \text{or} \quad \frac{12}{\cdot}=\frac{\cdot}{18}$$

We should probably think of the 6 days in connection with the 12 days, thereby placing the 6 in the same fraction with the 12, but, arithmetically, it does not now matter in which of the two fractions we place the 6, as the solution to any one of these four equations

$$\frac{18}{?}=\frac{6}{12} \qquad \frac{18}{6}=\frac{?}{12} \qquad \frac{12}{6}=\frac{?}{18} \qquad \frac{12}{?}=\frac{6}{18}$$

is the same; namely

$$6? = 12\times18$$

This would give us 36 as the number of men to be employed in doing the work in 6 days. However, we may simplify our problem still further in this instance by reducing the fraction in which 6 occurs, getting

$$\frac{18}{?}=\frac{1}{2} \qquad \frac{3}{1}=\frac{?}{12} \qquad \frac{2}{1}=\frac{?}{18} \qquad \frac{12}{?}=\frac{1}{3}$$

and thus arriving directly at our solution as 36 (either 2×18 or 3×12, depending upon which equation we took). This simple bit of reasoning, then, will take the curse off the problem of forming the equation

for an inverse proportion. The "rule of three", celebrated in nursery rhyme because it "perplexes me", now ceases to be perplexing.

TEST YOUR KNOWLEDGE OF RATIO AND PROPORTION WITH THESE PROBLEMS

136 4160 has the same relationship to 3120 as 4 has to what number?

137 The speeds of airplane, train, and car, are 280 m.p.h., 56 m.p.h., and 42 m.p.h. Give the ratio of the speed in its lowest terms.

138 If a boat drifts down stream 48 miles in 12 hours, how far will it drift in 15 hours?

139 The wing span of a bomber is 120 ft. What will be the wing span of a scale model if the scale is 1:72?

140 A certain division contains 3,000 artillery, 15,000 infantry, and 1,000 cavalry. If each branch is expanded proportionally until there are in all 20,900 men, how many will be added to the artillery?

141 Two pulleys are connected by a belt. The smaller one runs at a speed of 750 rpm and the larger at 200 rpm. What is the ratio of their speeds?

142 If 20 tons of iron cost $225, what will 500 tons cost?

143 A ship has provisions to last her crew of 500 men for six months. The ship picks up 700 men from a torpedoed vessel. How long will the provisions last all of the men on board?

144 If a jeep used 15 gallons of gasoline in driving 255 miles, how much would it use for a trip of 425 miles?

145 A pursuit plane flies 90 m.p.h. faster than a bomber in still air. The bomber travels 42 miles while the pursuit plane travels 56 miles. Find the average speed of each aircraft.

$$\left(\text{Hint: } \frac{\text{Distance pur. pl. trav.}}{\text{Speed pur. plane}} = \frac{\text{Distance bomber trav.}}{\text{Speed of bomber}}\right).$$

146 If $3\frac{1}{2}$ tons of coal cost $52.50, how much will $5\frac{1}{2}$ tons of coal cost?

147 Some maps are so constructed that equal areas of the earth's surface are represented by equal areas on the map. If the island of St. Thomas (32 sq. mi.) is represented by a figure $8'' \times 10''$, what should the size of Puerto Rico (3435 sq. mi.) be if drawn to the same scale?

148 An aerial map of an area 19 miles by 26 miles is to be made. If the scale is to be 1 inch = 3 miles, what will be the dimensions of the map?

149 The weight of an object varies inversely as the square of its distance from the center of the earth. Taking the radius of the earth as 4,000 miles, what would be the weight of a twelve-ton airplane flying at an altitude of 5 miles?

150 The intensity of illumination varies inversely as the square of the distance from the source of light. What proportion of light do you "lose" when you move your book from 3 feet from a lamp to 6 feet?

151 The wind pressure upon a flat surface held perpendicular to the path of the wind varies directly as the square of the wind velocity. What is the ratio of the pressure on the windshield of an airplane going 200 miles per hour to that of an airplane going 320 miles per hour?

152 If 54 men can build a barracks in 12 days, how many men must a contractor hire in order to complete the job in 9 days?

· THE USE OF LOGARITHMS ·

By Reginald Stevens Kimball, Ed.D.

BY the use of logarithms, numbers may more easily be multiplied together, divided, or changed in power. While the complete exposition of the derivation of logarithms must be deferred until a later issue of PRACTICAL MATHEMATICS, until after the student has gained a working knowledge of algebra, we are presenting this article on the use of logarithms this early in our course because the reader who knows how to employ logarithms in the solution of problems will be enabled to save himself much wearisome and unnecessary computation.—*Editor.*

WHAT WE MEAN BY POWERS

When a number is multiplied by itself, we say that it is raised to a power. Thus 2×2 is called the square, or the second power of 2; $2\times2\times2$ is the cube, or the third power; $2\times2\times2\times2$ is the fourth power, etc. These are written for brevity's sake with small superior numbers (called exponents), as follows:

$2^2=4$	$2^3=8$	$2^4=16$	$2^5=32$	$2^6=64$	$2^7=128$	$2^8=256$,	etc.
$3^2=9$	$3^3=27$	$3^4=81$	$3^5=243$	$3^6=729$	$3^7=2187$	$3^8=6561$,	etc.
$5^2=25$	$5^3=125$	$5^4=625$	$5^5=3125$	$5^6=15625$	$5^7=78125$	$5^8=390625$,	etc.

For your own understanding of the way numbers expand when raised to higher powers, we suggest that you continue the expansion of both 2 and 3, carrying the process to 2^{20} and 3^{20}. Preserve the results of these calculations, as you will find them a handy reference.

Multiplying by means of exponents

By reference to this little table, the reader will easily see that 16, the fourth power of 2, multiplied by 8, the third power of 2, gives 128, the seventh power of 2. Notice what has happened to the exponents: we have multiplied 16 by 8 by adding the 4 and the 3 to get a 7 as our new exponent.

Similarly, 32 times 8 may be expressed thus:

$$32\times8=2^5\times2^3=2^{5+3}=2^8=256$$

$$9\times81=3^2\times3^4=3^{2+4}=3^6=729$$

$$27\times243=3^3\times3^5=3^{3+5}=3^8=6561$$

$$625\times3125=5^4\times5^5=5^{4+5}=5^9=1953125$$

Dividing by means of exponents

Reversing the operation, let us divide 256 by 32. Here we begin with the eighth power of 2, dividing it by the fifth power of 2. Our answer is 8, which is the third power of 2. We have subtracted the exponent of the divisor from the exponent of the dividend.

$$2^8 \div 2^5 = 2^{8-5} = 2^3 = 8$$

Similarly,

$$128 \div 16 = 2^7 \div 2^4 = 2^{7-4} = 2^2 = 8$$

$$729 \div 81 = 3^6 \div 3^4 = 3^{6-4} = 3^2 = 9$$

$$6561 \div 243 = 3^8 \div 3^5 = 3^{8-5} = 3^3 = 27$$

$$1953125 \div 3125 = 5^9 \div 5^5 = 5^{9-5} = 5^4 = 625$$

For numbers which are exact powers of 2 or 3, this simple system helps us greatly by simplifying the operation.

Here is a point at which the table of powers of 2 and 3 recommended on page 88 will be found a great convenience.

The powers of 10

Unfortunately, there are many numbers which are not exact powers of either 2 or 3. In casting about for some number to use as the base for this exponential system, mathematicians finally hit upon 10, which has a peculiar property, which may be noticed in this brief table:

$10^2 = 100$ $\quad 10^3 = 1000$ $\quad 10^4 = 10000$ $\quad 10^5 = 100000$ $\quad 10^6 = 1000000$, etc.

The zero power—Extending our rule concerning dividing by subtracting exponents, let us see what happens when we divide a number by itself.

$$2 \div 2 = 1. \qquad 2^1 \div 2^1 = 2^{1-1} = 2^0. \qquad \therefore 2^0 = 1$$

$$3 \div 3 = 1. \qquad 3^1 \div 3^1 = 3^{1-1} = 3^0. \qquad \therefore 3^0 = 1$$

$$10 \div 10 = 1. \qquad 10^1 \div 10^1 = 10^{1-1} = 10^0. \therefore 10^0 = 1$$

(and so for any other number)

Negative powers of 10—By a further extension of this system, we may express decimals also as powers of 10.

$$1 \div 10 = 10^0 \div 10^1 = 10^{0-1} = 10^{-1} = 0.1$$

$$1 \div 100 = 10^0 \div 10^2 = 10^{0-2} = 10^{-2} = 0.01$$

WHERE WE GET LOGARITHMS

The logarithm is really the exponent of 10, indicating the power to which 10 would have to be raised to produce a given number. Since 1 is 10^0 and 10 is 10^1, it is evident that the numbers between 1 and 10 must be greater than 10^0 but less than 10^1. Likewise, numbers between 10 and 100 must be greater than 10^1 but less than 10^2.

Fractional powers of 10

These numbers, it is evident, will have fractional exponents or exponents expressed as mixed fractions. Since in fractional form they would present difficulties in addition or subtraction (because of the necessity of arriving at a common denominator), it is best to express them as decimal fractions. Using this system, we may say

$$2=10^{0.30103}$$
$$3=10^{0.47712}$$
$$4=10^{0.60206}$$
$$5=10^{0.69897}$$
$$6=10^{0.77815}$$
$$7=10^{0.84510}$$
$$8=10^{0.90309}$$
$$9=10^{0.95424}$$

To multiply 2 by 3, then, we may say

$$2\times3=10^{0.30103}\times10^{0.47712}=10^{0.30103+0.47712}=10^{0.77815}=6$$

Similarly,

$$8\div2=10^{0.90309}\div10^{0.30103}=10^{0.90309-0.30103}=10^{0.60206}=4$$
$$9\div3=10^{0.95424}\div10^{0.47712}=10^{0.95424-0.47712}=10^{0.47712}=3$$

Since, in all of these expressions, we have taken the numbers to the base 10, we may save ourselves a bit of trouble by not writing the 10's, using the exponents alone and adding or subtracting as the case may be. When we use them in this fashion, we call them logarithms (or, more familiarly, logs). In this form, our first example would be set up:

$$\begin{array}{lr} & \log 2=0.30103 \\ & +\underline{\log 3=0.47712} \\ \text{Then} & \text{antilog}\ \ 0.77815=6 \end{array}$$

(The expression, "the antilogarithm of a number", is usually written $\log^{-1}$.)

The second division example would read:

$$\begin{array}{lll} & \log & 9=0.95424 \\ & -\log & 3=\underline{0.47712} \\ \text{We find} & \log^{-1} & 0.47712=3 \end{array}$$

Tables of logarithms

In order to have these logarithms at hand when needed, mathematicians have computed the values of the logarithms of all numbers. It is simply necessary to refer to a table of logarithms to determine the correct logarithm for any number. In this issue of PRACTICAL MATHEMATICS (page 126), the reader will find a table of logarithms, correct to five places. By reference to this table, the reader will be saved much laborious figuring in connection with the solution of many of the problems which will be encountered in this and subsequent issues.

At first glance, a log table is a formidable-looking object, from which the reader makes little sense, since it seems to be merely a marshalling of rows of numbers. As these numbers take on meaning, however, you begin to appreciate their utility. As you use them, remember that you are the inheritor of the work of many mathematicians, extending over a long period of time. If you had to compute all of these logarithms before beginning to use them in your computations, you would find the process an endless one and would probably give up in the attempt. Since you have such a valuable tool at hand, it behooves you to learn how to use it as a means of saving time and labor.

THE PARTS OF A LOGARITHM

Before learning how to look up logs in the tables, the reader will need to consider one more property of numbers. We have previously noted that exponents of 10 for numbers between 1 and 10 may be expressed as a decimal fraction greater than 0 but less than 1.

The characteristic

In the same way, since logarithms of numbers between 10 and 100 are in excess of 10^1 but less than 10^2, it is evident that the number would be expressed as $10^{1+\textit{a decimal fraction}}$ and that exponents for numbers between 100 and 1000 would be expressed as $10^{2+\textit{a decimal fraction}}$, etc. In the same way, numbers smaller than 1 (that is, decimal expressions) would be expressed with a minus exponent to which the decimal fraction is added, numbers between 0.1 and 1 having an exponent greater than 10^{-1} but less than 10^0, those between 10^{-1} and 10^{-2} having an exponent greater than 10^{-2} but less than 10^{-1}, etc. When we use 10 as the base, we are greatly aided because the decimal fraction for the same combination of numbers is always the same. Thus,

our logarithms would be:

$\log 2 = 0.30103$	$\log 0.2 = -1 + .30103$
$\log 20 = 1.30103$	$\log 0.02 = -2 + .30103$
$\log 200 = 2.30103$	$\log 0.002 = -3 + .30103$

Negative characteristics—Because of the difficulty of writing expressions like those in the right-hand column above, we resort to one of two devices: (1) we may write the minus sign over the number which precedes the decimal (since this is the only part of the log which is negative), as $\log 9.2 = \bar{1}.30103$ (read "bar one"), or (2) we may subtract the 1 from 10, later subtracting 10 from the entire logarithm, as $\log 0.2 = 9.30103 - 10$, the form in which it is generally used in this country.

Determining the characteristic—The part of the logarithm which precedes the decimal point is called the characteristic. A positive characteristic tells how many digits appear in the number of which it is the logarithm. Note above: when the logarithm is 0, as in the case of 2, the number has one digit; when it is 1, the number has two digits; when it is 2, the number has three digits, etc.

For a positive characteristic, add 1 to determine the number of digits in the antilogarithm.

A negative characteristic tells how many zeros follow the decimal point before we get the first significant figure of the antilogarithm.

For a negative characteristic, subtract 1 to determine the number of zeros in the decimal antilogarithm.

TEST YOUR KNOWLEDGE OF CHARACTERISTICS WITH THESE EXERCISES

What is the characteristic of the logarithm of each of the following numbers?

1	4672	**5**	57937	**9**	0.21	**13**	0.000008
2	391	**6**	0.041	**10**	473962	**14**	0.020041
3	6	**7**	74	**11**	47.62	**15**	614
4	0.2	**8**	0.0076	**12**	516.37	**16**	0.003401

The mantissa

The part of the logarithm which follows the decimal point is called the mantissa. As we have seen, the mantissa is the same for all antilogarithms having the same sequence of numbers, regardless of where the decimal point appears in the antilogarithm.

HOW TO USE THE TABLES The tables on pages 126 and 127 enable us to find directly, correct to five places, the logarithms for all numbers having three significant figures. By a simple bit of calculation, we may utilize the tables in finding also the logarithms of numbers having four significant figures and may achieve approximate results for logarithms of numbers having more than four significant figures.

Let us consider first some simple cases, in which we make use of the tables exactly as they are before proceeding with the cases that demand a bit of calculation before the tables may be utilized.

Finding the logarithm

We look in column *N* for the first two digits and at the number in one of the columns across the top of the page for the third digit. The series of numbers opposite the first two digits and under the third is the mantissa of the logarithm desired. Turning to page 126, we find opposite *27* in column *0* the numbers 43136. Thus, we know that, for the sequence of figures, 270, the mantissa will always be 43136.

Combining this with our previous knowledge of characteristics, we are ready to say:

$$\begin{aligned}
\log 270 &= 2.43136\\
\log 27.0 &= 1.43136\\
\log 2.7 &= 0.43136\\
\log 0.27 &= 9.43136-10\\
\log 0.027 &= 8.43136-10\\
\log 0.0027 &= 7.43136-10, \text{ etc.}
\end{aligned}$$

To read the logarithm of the third digit when it is other than 0, we look in the appropriate column.

TEST YOUR KNOWLEDGE OF MANTISSAS WITH THESE EXERCISES

Referring to the tables of logarithms of numbers, find the mantissa of each of the following numbers:

17	320	**20**	67	**23**	1280	**26**	16
18	570	**21**	100	**24**	93	**27**	156000000
19	9	**22**	3.79	**25**	14200	**28**	42.8

Illustrative Problem on Multiplication

Multiply 12.7 by 9.82, making use of logarithms.

We first determine that the characteristic of 12.7 will be greater than 10^1 but less than 10^2. Its characteristic, then, will be 1. Similarly, the logarithm

of 9.82 will be greater than 10^0 but less than 10^1; it will have the characteristic of 0.

To find the mantissa (the decimal part) of the logarithm of 12.7:

a Move the index finger of the left hand down the *N*-column of the table of logarithms of numbers (page 126) until it is under 12.

b Move the finger along the horizontal to the *7*-column. The figures in the *7*-column are 10380. Combining this with our previously determined characteristic gives us

$$12.7 = 10^{1.10380}$$

The reading for 9.82 is made in the same way, as follows:

c Index finger under 98 in the *N*-column.

d Index finger moved to right along horizontal to the *2*-column. Here we read 99211. Combining this mantissa with the characteristic already determined, we have

$$9.82 = 10^{0.99211}$$

Observe that the mantissa was read in both cases in the column headed by the third figure of the number, regardless of where the decimal point in the original number was placed, and in the same line with the first two figures of the number as indicated in the *N*-column. Readings are always made in the same way, regardless of whether a number has three significant figures, or fewer than three, or more than three, and regardless of where the decimal point is located (since the characteristic takes care of the size of the number).

Thus,

$$\begin{aligned} 576 &= 10^{2.76042} \\ 57.6 &= 10^{1.76042} \\ 5.76 &= 10^{0.76042} \\ 0.576 &= 10^{-1.76042} \\ 0.0576 &= 10^{-2.76042} \end{aligned}$$

When a number has fewer than three significant figures, enough zeros are annexed to make three figures. Thus, 31 is read 31.0 and 7 is read 7.00 and we look for the mantissas in the *0*-column opposite 31 and 70, respectively.

When a number has more than three significant figures, the reading is taken for the first three only, and additions are made to the reading as explained in the paragraph on proportional parts, below.

We are now ready to proceed with our multiplication:

e $12.7 \times 9.82 = 10^{1.10380} \times 10^{0.99211} = 10^{1.10380+0.99214} = 10^{2.09594}$

The characteristic, 2, tells us that we shall have three digits before the decimal point in our answer. Since the table we are using could give us this information directly only in case the number desired were an integer (or had no more than three significant figures), we shall lay the problem aside temporarily while we learn how to find the antilogarithms of numbers in cases where the exact logarithms are not given in the table.

Interpolation for proportional parts

When we have more than three significant figures in the number for which we desire to find the logarithm, we must find the proportional part of the difference between the logarithms shown in the table. This is a simple arithmetic process which need not occasion us any consternation.

To find the logarithm of 792.7, we get, from the table on page 127, looking for 79 at the left under *N* and for *2* at the top, the log of 792 (which is 2.89873), obviously smaller than the desired log. We note that the log of 793 would be 2.89927, which in turn is larger than the desired log. The log which we want is $\frac{7}{10}$ of the difference between these two. Subtracting 89873 from 89927 gives us 54 as this difference. We next get $\frac{7}{10}$ of 54, or 37.8, or 38, which we add to the smaller mantissa, 89873, thus getting 89911 as the mantissa for 792.7, the complete log being (with the characteristic restored) 2.89911.

Computation

$$\begin{array}{rr} \log 793.0 = & 2.89927 \\ -\log 792.0 = & -2.89873 \\ \hline \text{dif}\quad 1.0 = & 54 \end{array} \qquad \frac{7}{10} = \frac{?}{54}$$

$$10? = 378$$

$$? = 37.8 \text{ or } 38$$

$$\begin{array}{rr} \log 792.0 = & 2.89873 \\ + \qquad .7 = & + \quad 38 \\ \hline \log 792.7 = & 2.89911 \end{array}$$

For a five-digit number, we follow the same procedure, except that we base our difference on hundredths instead of on tenths.

The process of finding the logarithm for 653.39 would be:

Computation

$$\begin{array}{rr} \log 654.00 = & 2.81558 \\ -\log 653.00 = & -2.81491 \\ \hline \text{dif}\quad 1.00 = & 67 \end{array} \qquad \frac{39}{100} = \frac{?}{67}$$

$$100? = 2613$$

$$? = 26.13 \text{ or } 26$$

$$\begin{array}{rr} \log 653.00 = & 2.81491 \\ + \qquad .39 = & + \quad 26 \\ \hline \log 653.39 = & 2.81517 \end{array}$$

TEST YOUR KNOWLEDGE OF INTERPOLATION WITH THESE EXERCISES

Referring to the tables of logarithms of numbers, find the mantissas of each of the following. (Solve the fifth digit by interpolation.)

29 64972	**32** 14.761	**35** 1725300
30 58959	**33** 298.36	**36** 768970
31 47891	**34** 8392.4	**37** 0.12345

Supplying the characteristics of each of the above, complete the logarithm.

How to read an antilogarithm

We have previously seen that the mantissa for any numbers comprised of the same sequence of digits will always be the same, the position of the decimal point in this sequence being indicated by the characteristic (the number preceding the decimal point in the logarithm). When we do not find a mantissa in the table, we take the next smaller mantissa which does appear and compute from that.

Given the logarithm, 7.72659−10, we reason:

The mantissa, 72659, does not appear in the tables. The next smaller mantissa which does appear, 72591, is for the number sequence 532 (found by reading 53 at the left under *N* and *2* at the top of the column). This is 68 less than the given mantissa. The difference between this mantissa and the next in the table (for 533), 72673, is 82. The number which we desire, then, lies $\frac{68}{82}$ of the way between 532 and 533. We may set this up as a proportion or work it out as a decimal fraction, getting the same results in either case, 828 or 83, depending on the number of places to which we wish to carry our computation. Annexing these digits to the number sequence, 532, gives us the sequence, 532828 or 53283. The characteristic, −3 (obtained by subtracting 10 from 7), indicates that there will be two zeros following the decimal point before we write the significant figures (see rule above, page 92). Hence, the desired antilogarithm is 0.00532828 or 0.0053283.

Computation

Required, the antilogarithm of 7.72659−10:

$$\begin{array}{rlcr} & \text{man } 72659 & = & ? \\ - & \underline{\text{man } 72591} & = & \underline{-532000} \\ & \text{dif} \quad 68 & = & ? \end{array} \qquad \frac{68}{82}=\frac{?}{1000} \quad \text{or} \quad \frac{68}{82}=\frac{?}{100}$$

$$?=828 \text{ or } 83$$

$$\begin{array}{rlcr} & \text{man } 72673 & = & 533000 \\ - & \underline{\text{man } 72591} & = & \underline{-532000} \\ & \text{dif} \quad 82 & = & 1000 \end{array} \qquad \begin{array}{rlcr} & \text{man } 72591 & = & 532000 \\ + & \underline{\text{dif} \quad 68} & = & \underline{+ \quad 828} \\ & \text{man } 72659 & = & 532828 \end{array}$$

$$\log^{-1} 7.72659-10=0.00532828$$

Completing the Illustrative Problem on Multiplication

Returning to our problem of multiplying 12.7 by 9.82, we have already seen that we may express them as $10^{1.10380} \times 10^{0.99214} = 10^{2.09594}$.

Looking up 09594 in the table of logarithms of numbers (page 126), we do not find exactly that, but we do find, opposite *12* under the *N*-column in column *5* 09691 and in column *4* 09342. The difference between these two is 349. The difference between the mantissa of our actual log, 09594, and the next smaller in the table, 09342, is 252. To the number sequence, 124, then, we want to annex $\frac{252}{349}$, or .722. This gives us the number sequence, 124722. The characteristic, 2, tells us that we shall have three figures before the decimal, this giving us, for the number sequence found, 124.722. By actual multiplication, we get $12.7 \times 9.82 = 124.714$; hence, our answer, found by logarithms, is only 0.008 out of the way. Since our table does not guarantee us accuracy past the fourth figure, we are sufficiently accurate.

If we desire greater accuracy, we may use logarithmic tables which permit the computation to more places. For this purpose, PRACTICAL MATHEMATICS recommends Marsh's *Interpolated six-place tables of the logarithms of numbers* (New York: Wm. H. Wise & Co., Inc., 1942) or Clark's *The slide rule and logarithmic tables* including a ten-place table of logarithms (Chicago: Frederick J. Drake & Co., 1941).

Computation

$$\begin{array}{r} 12.7 \\ \times 9.82 \\ \hline 254 \\ 1016 \\ 1143 \\ \hline 124.714 \end{array}$$

$$\begin{array}{rr} \log 12.7 = & 1.10380 \\ +\log 9.82 = & +0.99214 \\ \hline \log \ ? & 2.09594 \end{array}$$

$$\begin{array}{lr} \log^{-1} 09691 = & 125000 \\ -\log^{-1} 09342 = & -124000 \\ \hline \text{dif} \quad 349 = & 1000 \end{array}$$

$$\begin{array}{lr} \log^{-1} 09594 = & ? \\ -\log^{-1} 09342 = & -124000 \\ \hline \text{dif} \quad 252 = & ? \end{array} \qquad \frac{252}{349} = \frac{?}{1000}$$

$$\begin{array}{lr} \log^{-1} 2.09342 = & 124.000 \\ +\text{dif} \quad 252 = & +\quad 722 \\ \hline \log^{-1} 2.09594 = & 124.722 \end{array} \qquad ? = 722$$

TEST YOUR KNOWLEDGE OF MULTIPLICATION WITH THESE EXERCISES

Multiply, using logarithms:

38 27×89
39 732×658
40 12.7×6.54
41 0.0179×0.363
42 2.19×63.8
43 65.94×392.7
44 537.6×0.04834
45 37.68×43.23
46 6.078125×89.0265

Illustrative Problem on Division

Divide 124.722 by 9.82.

In dividing, we subtract the logarithm of the divisor from the logarithm of the dividend.

We determine the logarithms of the two numbers just as we did in the case of multiplication (page 93). Since the numbers used in this example are the same as those used in the illustration for multiplication, we may refer to our previous computations for the necessary data.

$$\begin{array}{rl} \log 124.722 = & 2.09594 \\ -\log \quad 9.82 \ = & -0.99214 \\ \hline \log \quad ? \quad = & 1.10380 \end{array}$$

Looking up 10380 in the table, we find it in column *7* in line with the figures 12. Hence, our number sequence is 127. The characteristic, 1, indicates that there will be two digits before the decimal. Our answer, then, is 12.7.

TEST YOUR KNOWLEDGE OF DIVISION WITH THESE EXERCISES

Divide, using logarithms:

47 $19\overline{)323}$
48 $260\overline{)33800}$
49 $3700\overline{)4810000}$
50 $9.7\overline{)48.5}$
51 $0.379\overline{)65}$
52 $0.0683\overline{)7.941}$
53 $42.86\overline{)143.07}$
54 $173.39\overline{)672}$
55 $64.293\overline{)2063.71}$
56 $0.473926\overline{)5.69372}$
57 $0.0019437\overline{)69.842}$
58 $65.847\overline{)0.0032548}$

Using cologarithms

In many instances, division is more easily accomplished by the use of the cologarithm. A cologarithm, sometimes called the arithmetical complement of a number, is found by subtracting the logarithm of the number from 10 − 10. When we use the cologarithm of the number in division examples, we add the cologarithm instead of subtracting the logarithm of the divisor.

In ordinary division, it is just as easy to make use of the rule we have previously learned. When we have a series of multiplications and divisions to perform at one time, however, employment of the cologarithms of the terms by which we are to divide makes it possible for us to combine the logarithmic values in one operation.

Illustrative Problem on Cologarithms

Find the cologarithm of 716.

By reference to the table on page 127, we find opposite 71 in the *6*-column the figures, 85491. Since 716 contains three digits, we know that its charac-

teristic will be 2. Hence, the logarithm of 716 is 2.85491. Subtracting this from 10−10 gives us 6.14509, the cologarithm of 716.

Computation

$$\begin{array}{rl} & 10.000000-10 \\ \log 716 = - & \underline{3.85491} \\ \text{colog } 716 = & 6.14509\ \ -10 \end{array}$$

TEST YOUR KNOWLEDGE OF COLOGARITHMS WITH THESE EXERCISES

Find the cologarithms of each of the following:

59 327	**62** 84.72	**65** 6.2764
60 4.82	**63** 163.2	**66** 0.49527
61 59.6	**64** 9.473	**67** 0.0046298

USE OF LOGARITHMS IN CHANGING POWERS

As was mentioned in our introductory paragraph, one of the most advantageous uses of logarithms occurs when we find it necessary to extract a root of a given number or to raise the number to a power. While either of these processes may be performed arithmetically, we find it much easier to make use of our knowledge of logarithms in performing the necessary operation.

Raising a number to a higher power

In multiplying two numbers, the reader will remember, we add the logarithms of the two numbers. In multiplying a number by itself, accordingly, we add its logarithm to its logarithm, which is equivalent to multiplying the number by 2.

Illustrative Example

$$28.2^2 = 28.2 \times 28.2$$

$$\begin{array}{rr} & \log 28.2 = \quad 1.45025 \\ + & \underline{\log 28.2} = \underline{+1.45025} \\ & \log \quad ? \ = \quad 2.90050 \end{array} \qquad \begin{array}{rr} & \log 28.2 = 1.45025 \\ \times & \underline{\qquad 2} \quad \underline{\times\ 2} \\ & \log \quad ? \ = 2.90050 \end{array}$$

$$\begin{array}{r} 28.2 \\ \times \underline{28.2} \\ \underline{564} \\ 2256 \\ \underline{564} \\ 795.24 \end{array}$$

$$\begin{array}{rll} & \log^{-1} 90091 = & 796000 \\ - & \underline{\log^{-1} 90037} = & \underline{-795000} \\ & \text{dif} \qquad 54 & \quad 1000 \end{array} \qquad \frac{13}{54} = \frac{?}{1000} \qquad ? = .240$$

$$\begin{array}{rll} & \log^{-1} 90050 = & \quad ? \\ - & \underline{\log^{-1} 90037} = & \underline{-795000} \\ & \text{dif} \qquad 13 & \quad ? \end{array} \qquad \begin{array}{rll} & \log^{-1} 90037 = & 795000 \\ + & \underline{\text{dif} \qquad\qquad 13} = + & \underline{\quad 240} \\ & \log^{-1} 90050 = & 795.24 \end{array}$$

Following the same line of reasoning, we see that we may obtain the third power of a number by multiplying the log of the number by 3, the fourth power by multiplying the log by 4, etc.

Extracting the root of a number

Referring to the previous example, we note that the log of 28.2 is exactly half of the log of its square, 795.24. Since 795.24 is the square of 28.2, it follows that 28.2 is the square root of 795.24. To extract the square root of a number, then, divide its log by 2 and find the antilogarithm of the resultant logarithm.

Similarly, the cube root of a number may be found by dividing the log of the number by 3, the fourth root by dividing the log by 4, etc.

TEST YOUR KNOWLEDGE OF POWERS WITH THESE EXERCISES

Perform the indicated operations, using logarithms:

68 67^3	**71** 179.5^6	**74** $\sqrt[5]{683.6}$
69 483^2	**72** 3.2794^4	**75** $\sqrt[3]{3.7348}$
70 21.47^4	**73** $\sqrt{743}$	**76** $\sqrt{647.253}$

COMBINING OPERATIONS Before leaving the study of the logarithms of numbers, let us take an example which calls for the use of several of the steps which we have learned, all in one operation. This will serve to show the advantage of using cologarithms and will also serve as a general review for the entire subject.

To get the value of

$$\frac{184.3 \times 26.71}{\sqrt{169.27}} \times \frac{\sqrt[3]{172.83}}{491.54}$$

we may set up our framework as follows:

$$\begin{aligned}
&\log 184.3 = ?\\
&\log 26.71 = ?\\
&\tfrac{1}{2} \text{ colog } 169.27 = ?\\
&3 \times \log 172.83 = ?\\
&\text{colog } 491.54 = ?
\end{aligned}$$

The solution is left to the student.

The real value of this part of the study will come when the student applies it to the solution of the numerical exercises which follow in other portions of PRACTICAL MATHEMATICS and in his daily use of logarithms as time-saving devices in any problem which calls for multiplication, division, or changing powers of involved numbers. Resolve now to get the requisite practice, as only in this fashion will you get past a theoretical knowledge of the subject

· THE SLIDE RULE ·

By Robert N. Farr, Sc. M.

TO save time and effort in performing complicated operations in multiplication, division, roots, and powers, mathematicians have devised an instrument known as the slide rule. The use of the slide rule is not difficult; anyone with average intelligence can learn to use it in a very short time.

Before undertaking the study of its use, however, you should secure a slide rule of your own, as you will find it difficult to understand the instructions unless you practice each step on the rule. Fairly satisfactory slide rules may be purchased at five- and ten-cent stores, as well as at most stationers' and school-supply dealers', at a cost of from twenty-five cents to one dollar.

In reading this article, you should have the slide rule conveniently at hand and should follow each step of the explanation by manipulating your rule according to the printed instructions. In a classroom, your instructor would demonstrate one step at a time and would check on your understanding of that point before he proceeded with his explanation of the next; since you are working by yourself, it is more than essential that you make sure of the extent of your understanding before you try to forge ahead with the reading.

HOW TO USE THE SLIDE RULE

To the uninstructed student, the slide rule may appear difficult to understand because of the numerous scales which are printed on it. In reality, however, the operation is comparatively simple. Once the general principles are understood, success in its use depends upon the degree of facility which the operator acquires through practice.

The parts of the rule

The slide rule consists of three parts: the *rule* itself; the *slide*, which moves in the grooves of the rule; and the *runner*, sometimes called the *indicator*. The runner is a piece of glass or plastic, mounted in guides, by means of which it slides over the face of the rule. On the under-side of the glass is etched a hairline, which is used for accuracy in setting and marking results.

Understanding the scales

The face of the rule has several scales marked upon it. For convenience, these scales are designated by letters. All slide rules following the Mannheim type have scales marked *A*, *B*, *C*, and *D*. Those rules which follow the polyphase type have also a *CI*- and a *K*-scale. In the discussion in this article, we shall limit our consideration to these six scales.

The slide rule pictured below is similar to most of the slide rules obtainable on the market. Rules prepared by the various supply houses differ slightly in arrangements and markings, but possess the same general appearance and are manipulated in the same manner as this one.

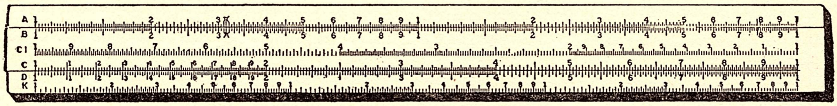


Fig. 1

Readers who have need for more complicated slide rules, with special scales, will be able to make the transition to other scales readily by mastering the principles given here and then attending to the special instructions provided in the manual which accompanies the rule. Be sure to secure the manual at the time you purchase the rule.

The general explanations which are printed in this article apply to almost all types of slide rule now on the market. While the diagrams are confined to the straight-line type of rule, the explanations may be transferred quite readily to the circular type of slide rule, since that type, also, makes use of logarithmic scales.

DERIVATION OF THE SCALES

Each scale on the slide rule is a mechanical representation of the mantissas of logarithms. The slide rule is simply a device for enabling its user to perform the addition of logarithms without being put to the necessity of performing all of the computations set forth in the preceding article. (See pages 88 to 100.)

On each scale of the slide rule, the vertical line at the extreme left (corresponding to zero on the ordinary ruler or yardstick) is marked 1, and is called the *left index*. At the right end of each scale will be found another vertical line marked 1, called the *right index*.

The distance between 1 and 2 on the slide-rule scale is a little more than 30 per cent of the whole length of the scale. (Remember that the logarithm of 2 was found to be 0.30103.) The distances between 2 and 3, 3 and 4, and so on, become smaller and smaller, just as the differences between the logs of these numbers decrease as we ascend the scale.

MEASURING THE SCALES

For the purpose of securing even more accurate readings, the distances between the numbers are marked off into tenths (these again decreasing in size as we go up the scale). On the *C*- and *D*-scales, each of these tenths is numbered, for ease in reading, and then subdivided again into tenths.

READING THE SCALES

In finding the mantissa of a number on the slide rule, we must take into account the various subdivisions of the rule. Just as with logarithms, the slide rule presents the same reading for a given sequence of numbers without regard to where the decimal point appears in the sequence.

This means that 16, 18.2, 1739, and 1.49 are all located in the same general area on the scale, between 1 and 2. In determining the accurate settings, however, we must consider the numbers which follow the one.

The whole number, 16, is read as "unit 1 plus 6 major divisions". On the *C*- or *D*-scale, this would be at the small 6 between the large 1 and the large 2. On other scales, it would be at the sixth major division (unnumbered), between 1 and 2.

The number, 18.2, is read as "unit 1 plus 8 major divisions plus 2 minor divisions". It is important to remember that we always disregard the decimal point when locating a number on the scale.

The number, 1739, presents a further difficulty, as the ten-inch slide rule is not accurate past the third figure. We use unit 1 again, and locate the seventh major division, following the same line of reasoning that we used in the previous example. This seventh major division is divided into ten minor divisions. We take three of these, to represent the 3 in the 1739, and then set the hairline nine-tenths of the distance between the third and the fourth minor divisions. (This, of course, is almost on the fourth minor-division line.)

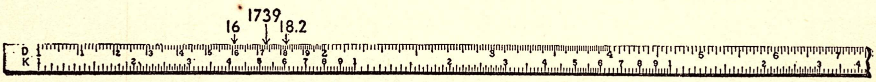


Fig. 2

Readings for 16, 18.2, and 1739 are shown in Fig. 2. After finding these readings on your own rule, try to find 1.49.

Unit 2 is the next main division on the scale. As in the case of unit 1, it is divided into ten major divisions. Each of these major divisions is divided into 5 minor divisions, each of which is one-fifth (or two-tenths) of the major division, or two one-hundredths of unit 2. Every number beginning with digit two falls within unit 2. The number 21.6, therefore, is read as "2 plus 1 major division plus 3 minor divisions of the following major division" (remembering that each minor division represents 0.2).

The number, 0.263, is read as "unit 2 plus 6 major divisions plus $1\frac{1}{2}$ minor divisions of the following major division".

Unit 3 is divided in the same way as unit 2 and the divisions have the same values.

Unit 4 is divided into ten major divisions, each of which has two minor divisions with values of five-tenths of the major division or 0.05 of the unit 4. When we have a three-digit number in this unit, the third digit must be estimated, unless it happens to be 5.

Suppose, for example, we try to locate the number, 435, on scale *D*. 435 is read as "4 plus 3 major divisions plus one minor division". (See Fig. 3.) The remaining units on the *D*-scale are exactly like unit 4, except that the division lines are drawn closer together.

Try finding various number sequences on your own slide rule. Remember that the location of the decimal point is of no significance in connection with slide-rule readings. The numbers, 6.2, 620, and 6200 are all found at the same spot as is 62. Can you locate it?

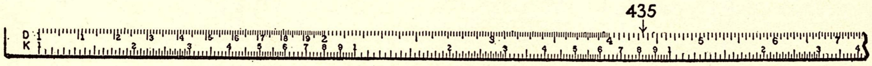


Fig. 3

Before beginning to use the slide rule, you should know which scales are used to perform the different operations. Until you are thoroughly familiar with the operation of the slide rule, it would be wise to refer frequently to the following table:

Operation	Use Scales
Multiplication	*C* and *D* (or *A* and *B*)
Division	*C* and *D* (or *A* and *B*)
Squaring a number	*A* and *D*
Square root	*A* and *D*
Cubing a number	*K* and *D*
Cube root	*K* and *D*
Reciprocal	*CI* (the *C*-scale in reverse)

In connection with all of the examples presented in Practical Mathematics from this point forward, we suggest that you secure practice on the slide rule by using the instrument in any numerical calculations called for. Reference to the table above will help you to determine quickly which scales on the rule are best suited for the particular operation which you are called upon to perform.

A slide rule which merely occupies space in the pocket or in the desk drawer may serve some good purpose in impressing your friends with your profound knowledge of mystical tools, but it is doing you little actual good. Resolve now to utilize this time-saving instrument so that you may have more time available for acquiring additional knowledge.

USING THE SLIDE RULE FOR MULTIPLICATION

When we multiply by means of logarithms, we add together the logs of the numbers which are to be multiplied. When we use the slide-rule representations of logarithms, we do the same thing mechanically. To multiply two numbers on the slide rule, we place the distances representing their logarithms end to end. The total length is the mantissa of the logarithm of the product of the numbers.

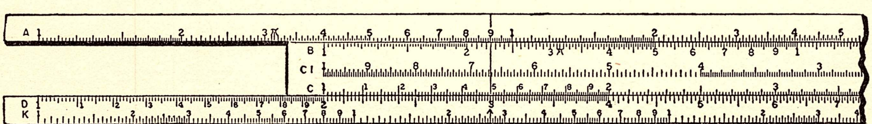
Since the *C*- and *D*-scales are exactly alike, we find the log of one number on one scale and the log of the other number on the other scale. Since these scales are movable, we can slide them along until these two logs lie in a straight line. (The *A*- and *B*-scales may also be used for any operation in which these directions call for the use of the *C*- and *D*-scales, but, since the *C*- and *D*-scales are longer, and the distances between units consequently greater, they afford greater accuracy than do the shorter scales.) Try each of the following problems first on the *C*- and *D*-scales and then on the *A*- and *B*-scales.

Illustrative Problem A

Multiply $1\frac{1}{2}$ by 2 on the slide rule.

Steps

a We locate $1\frac{1}{2}$ (thinking of it as 1.5) on the *D*-scale at the fifth major division after the 1 at the left.



$1\frac{1}{2} \times 2 = 3$

Fig. 4

b Over this, we set the left index of the *C*-scale.
c We locate 2 on the *C*-scale at the large 2 representing the second unit.
d We slide the runner along until the hairline is over this 2.
e Directly below the hairline, on the *D*-scale, we read the number 3, thereby concluding that 3 is the answer for our problem in multiplication.

With the rule in its present setting, you are able to read the result of multiplying any number by 2.

If either of the numbers to be multiplied is greater than 5, we employ the right index instead of the left index.

Illustrative Problem B

Using the slide rule, multiply 3.4 by 7.2.

Steps

a Set the right index of the *C*-scale over 7.2 on the *D*-scale.
b Move the indicator until the hairline of the glass indicator is over 3.4 on the *C*-scale.
c The answer, 24.5, is read under the hairline on the *D*-scale.

The easiest way in which to locate the decimal point in a problem such as the above is to multiply mentally the first digits of the factors. When this is done, place the decimal point in the result so that the value is nearest to that of the mental multiplication. In this case, 7×3 equals 21, the result on the *D*-scale is 245; we are accurate when we place the decimal point so as to make the result read 24.5, since 24 is nearer to the determined value of 21 than is 2.45 or 245.

Illustrative Problem C

Using the slide rule, multiply 18.72×0.356.

Steps

a Set the left index of scale *C* over 1872 on the *D*-scale.
b Then move the indicator until the hairline is over 356 on the *C*-scale.
c Read the answer, 666, on the *D*-scale under the hairline.
d By mental multiplication, we determine that the most logical answer (and the correct answer) is 6.66.

Application A

How much would it cost to ship a box weighing 125 pounds from New York to Philadelphia if the cost per pound is 4.5 cents per pound?

Solution

Using the slide rule as directed above, we find that 125×0.045 equals 5.63; therefore, the cost for shipping may be determined as $5.63.

Application B

A man earning $87\frac{1}{2}$ cents an hour worked for $7\frac{3}{4}$ hours a day for $11\frac{1}{2}$ days. How much did he earn?

Solution

After setting the slide rule for 875×775, set the indicator over the answer on *D*; treat this product as the first factor. Set 1 on *C* to the hairline. Read the answer on *D* under 115 on *C*.

TEST YOUR ABILITY TO USE THE SLIDE RULE FOR MULTIPLICATION WITH THESE EXERCISES

1 15×19

2 $2.7 \times \times 11.1$

3 90×8.1

4 44.3×18

5 A voltage of 110 volts is impressed across a lamp and a current of 0.52 amperes flows through it. What is the wattage? (To find wattage, multiply volts by amperes).

USING THE SLIDE RULE FOR DIVISION

In division, we use scales *C* and *D* in the reverse order from that in which we used them for multiplication. When performing the operation of division, we place the hairline of the glass indicator over the number to be divided on the *D*-scale, moving the slide to the right or to the left to place the number by which we are dividing (which appears on the *C*-scale) directly under the hairline. The solution will be found under the index of the *C*-scale on the *D*-scale. For an illustration, let us take a simple problem to which we already know the answer:

Illustrative Problem

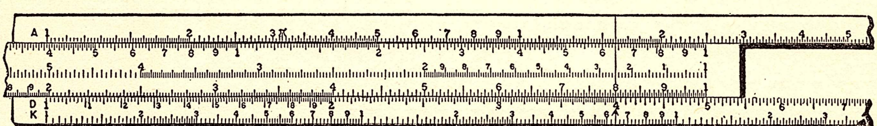
$$\frac{8}{4}=2$$

Solution

a Set the hairline of the glass indicator over 8 on the *D*-scale.

b Move the slide until 4 on the *C*-scale is directly under the hairline.

c The answer, 2, is read under the left index of the *C*-scale on the *D*-scale. See Figure 5.



$\frac{8}{4}=2$

Fig. 5

The rules for approximating the decimal point in division are the same as those in multiplication.

TEST YOUR ABILITY TO USE THE SLIDE RULE FOR DIVISION WITH THESE EXERCISES

6 $20 \div 4$

7 $122 \div 2$

8 $29.97 \div 2.7$

9 $983.2 \div 47.3$

10 If the average marching speed of a company of soldiers is 3.25 miles per hour, in how many hours will they cover 15 miles? (Hint: Divide distance by rate to get time.)

11 There are 125 workers in a small machine shop. The sum of their ages is 4500. What is the average age?

USING THE SLIDE RULE FOR SQUARING A NUMBER

We have already learned that, when we speak of squaring a number, we mean multiplying the number by itself, that 3^2 is just 3×3. In squaring a number, then, we may proceed as in ordinary multiplication, finding the number once on the *C*-scale and again on the *D*-scale (or on the *A*- and *B*-scales).

There is a simpler way, however. Since the *A*-scale is one-half as long as the *D*-scale, the numbers on it mount up faster than on the *D*-scale. We get the same results on the slide rule by the combination of scales *A* and *D* that we secured in the use of logarithms when we multiplied

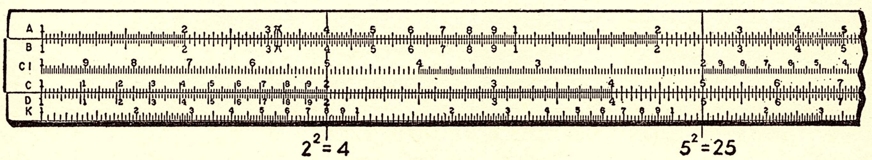


Fig. 6

a log by 2. Every number on the *D*-scale is directly below its square on the *A*-scale. All that we have to do is to place the hairline of the indicator on the number which we are squaring, look directly above, to the *A*-scale, and read our answer without moving the slide at all.

The simplicity of this operation is likely to mislead the reader into thinking that there is "nothing to" the task of finding the square by means of the slide rule. When one is confronted by the problem of finding the square of numbers containing three or more digits, however, the difficulty is compounded because it is impossible, on a 10-inch slide rule, to secure accuracy in the fifth and sixth figures. We can only estimate approximately the desired result.

In squaring 794, for example, we may estimate that the square will be a little less than 800^2, or 640,000. We know also, by inspection, that the square of 794 will end in 6, since $4 \times 4 = 16$. Try this combination on your rule and compare your findings with the actual result obtained by multiplying 794 by 794.

Illustrative Problem A

$2^2 = 4.$

Steps

a Set the hairline of the indicator over 2 on the *D*-scale.
b Read 4 under the hairline on the *A*-scale.

Illustrative Problem B

Square 18, using the slide rule.

Steps

a Make the setting as explained above.
b We know that $10^2=100$ and that $20^2=400$, and since 18 lies between 10 and 20 but is nearer to 20, the answer, we surmise, will be near to 400.
c Therefore, we read our solution as 324.

TEST YOUR ABILITY TO USE THE SLIDE RULE FOR SQUARING A NUMBER WITH THESE EXERCISES

12 Square 6, using the slide rule.
13 What does 40^2 equal?
14 Square 0.753.
15 What does 34.4^2 equal?
16 Square 613.4
17 Find the square of 7.32

FINDING THE SQUARE ROOT

When we attempt to determine the square root of a number, we are looking for a number which, when multiplied by itself, will be equal to the given number. For this operation, we again employ the *A*- and *D*-scales of the slide rule. Before we can begin the operations with the slide rule, we must first mark off the number into groups of two digits each, beginning at the decimal point. For example, 144, which is a whole number, is marked off into two groups with 44 in one group and 1 in the other in this manner: $\sqrt{1|44}$

In the answer to the square root of a number, there will be as many numbers in the answer as there are groups in the number itself. Finding the square root of a number calls for the reverse of the operation of squaring.

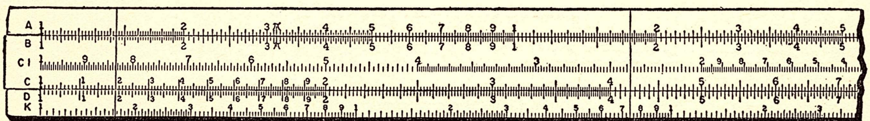


Fig. 7

If the first group has one number, as shown in $\sqrt{1|44}$, set the hairline of the glass indicator over the number on the left-hand side of the *A*-scale (called the *A′*-scale) and read the answer on the *D*-scale. If, however, the first group happens to have two numbers, as in the case of 21|65, set the hairline of the glass indicator over 2165 on the right half of the *A*-scale (called the *A″*-scale) and read the answer under the hairline on the *D*-scale.

Illustrative Problem A

Find the square root of 144.

Solution

a In this problem, we find one number in the first group.
b We therefore set 144 on the left half of the *A*-scale and read the answer, 12, under the hairline on the *D*-scale.
c As the number has two groups, we shall have two whole numbers in our answer.

Illustrative Problem B

Find the square root of 48.

Solution

a As we find that this group has two numbers, we set the scale on 48 in the right half of the *A*-scale.
b We read 693 on the *D*-scale.
c Here we have only one group; therefore, we have only one whole number and the result is 6.93.

Illustrative Problem C

Find the square root of 1296.

Solution

a In this problem, we find two numbers in the first group.
b Using the right half of the *A*-scale, we set 1296 (a very little short of 13, or 1300) on the scale and read the answer, 36, under the hairline on the *D*-scale.
c As the number has two groups, we shall have two whole numbers in our answer.

Illustrative Problem D

Find the square root of 7.

Solution

a Since there is only one number, we use the left half of the *A*-scale.
b Setting the hairline of the indicator over 7 on the *A′*-scale, we read the answer, 264, on the *D*-scale.
c As the number has only one group, we shall have one whole number in our answer. Pointing off, we thus get 2.64 for our final answer.

TEST YOUR ABILITY TO USE THE SLIDE RULE IN FINDING SQUARE ROOTS WITH THESE EXERCISES

18 Find the square root of 1382.
19 Find the square root of 12300.
20 Solve, using the slide rule, $\sqrt{625}$.
21 Find the square root of 21.100.
22 Find the square root of 1476.83.
23 What is the square root of 137.964?

CUBING A NUMBER

To cube a number, we multiply it by itself three times: thus, $3 \times 3 \times 3$ may also be written 3^3. To use the slide rule to cube a number, we employ the *D*- and *K*-scales, since the *K*-scale is one-third as long as the *D*-scale. First, we set the hairline of the glass indicator over the number to be cubed on the *D*-scale, and then read the answer directly on the *K*-scale, under the hairline. Upon a re-examination of the *K*-scale, you will note that there are four indexes dividing the scale into three parts. When the cube falls in the left third, the answer will be between 1 and 10 (Example: $2^3 = 8$). When the cube falls in the middle third, the answer will be between 10 and 100 (Example $4^3 = 64$). When the cube falls in the right third, the answer will be between 100 and 1000 (Example $8^3 = 512$). This may also be changed so that the left third includes numbers between 1000 and 10,000; the middle third, numbers between 10,000 and 100,000; and the right third, numbers between 100,000 and 1,000,000; etc.

Illustrative Problem A

What is 6 cubed?

Solution

a Set the hairline of the glass indicator over 6 on the *D*-scale.
b Note that the answer falls in the right third, where the numbers read from 100 to 999.
c Therefore, the answer on the *K*-scale under the hairline is read 216.

Illustrative Problem B

What is the cube of 14?

Solution

a Set the hairline of the glass indicator over 14 on the *D*-scale.
b Note that the answer falls in the left third, where the numbers read from 1000 to 9999.
c The answer on the *K*-scale under the hairline is a little less than 275. We know that the cube of 4 is 64, and thus see that our answer should end in 4. We take, therefore, 2744 for our final answer.

TEST YOUR ABILITY TO USE THE SLIDE RULE IN CUBING A NUMBER WITH THESE EXERCISES

24 Cube 12.
25 Cube .572.
26 Cube 81.5.
27 Cube 1.17.
28 Cube 14.63.
29 Cube 137.9.

FINDING THE CUBE ROOT

For this operation, we again employ the *K*- and *D*-scales. Before we can extract the cube root of a number, we must divide it into groups of three digits each, beginning at the decimal point just as we did in the case of the square root. In some cases, we shall have only one or two digits in the first group. In the case of 1|876.235, we have only one number in the first group; however, in the case of 11|865, we have two numbers in the first group. If the first group has only one digit, we set the hairline of the glass indicator over the number in the *left third* of the *K*-scale, and proceed to read the answer on the *D*-scale under the hairline. If the first group contains two numbers, we set

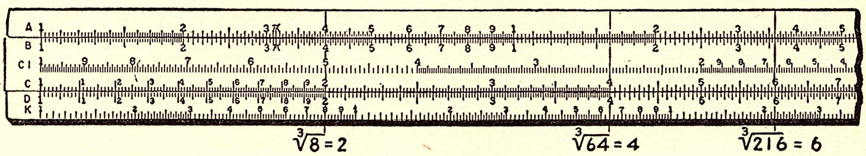


Fig. 8

the glass indicator over the number in the *middle third* of the *K*-scale and read the answer under the hairline on the *D*-scale. If the first group contains three numbers, set the hairline of the glass indicator over the number in the *right third* of the *K*-scale, and proceed to read the answer on the *D*-scale.

Illustrative Problem A

Find the cube root of 64.

Solution

a The first (and only) group contains two numbers; therefore, we set the hairline of the glass indicator in the middle third of the *K*-scale over 64.
b We read on the *D*-scale the answer, which is 4.

Illustrative Problem B

Find the cube root of 216.

Solution

a The first (and only) group contains three numbers; therefore, we set the hairline of the glass indicator in the right third of the *K*-scale over 216.
b We read on the *D*-scale the answer, which is 6.

TEST YOUR ABILITY TO USE THE SLIDE RULE IN FINDING CUBE ROOT WITH THESE EXERCISES

30 Find the cube root of 84.7.
31 Find the cube root of 125.
32 Find the cube root of 45.7.
33 Find the cube root of 753.6.
34 Find the cube root of 3130.
35 What is the cube root of 3.672?

USING RECIPROCALS

The reciprocal of any number is 1 divided by that number: therefore, the reciprocal of four is $\frac{1}{4}$, the reciprocal of 2 is $\frac{1}{2}$, and so on. The *CI*-scale on the slide rule will give us the reciprocal of any number on the *C*-scale. Upon examination, you will note that 25 on the *CI*-scale is directly above 4 on the *C*-scale. Since we know that the reciprocal of 4 is $\frac{1}{4}$, we can readily see that 0.25 is the reciprocal of 4, since 0.25 is just another way of writing $\frac{1}{4}$.

To demonstrate the practical use of the reciprocal scale, let us take some examples:

Illustrative Problem A

Multiply 25 by $\frac{1}{5}$ (the reciprocal of 5).

Solution

a We set the left index of the *C*-scale at 25 on the *D*-scale, as we should do in multiplication.
b We next move the glass indicator so that the hairline is above 5 on the *CI*-scale.
c We read the answer, 5, under the hairline on the *D*-scale.

Illustrative Problem B

Multiply 32 by $\frac{1}{8}$ (the reciprocal of 8).

Solution

a We set the left index of the *C*-scale at 32 on the *D*-scale, as we should do in multiplication.
b We next move the glass indicator so that the hairline is above 8 on the *CI*-scale.
c We read the answer, 4, under the hairline on the *D*-scale.

The *CI*-scale may also be used to multiply three factors together. Here is a typical example:

Illustrative Problem

Multiply $1.62 \times 0.7 \times 5.63$.

Solution

a Set the hairline of the glass indicator over 162 on the *D*-scale.
b Move 7 on the *CI*-scale under the hairline.
c Then slide the indicator so that the hairline is over 563 on the *C*-scale.
d The solution, read under the hairline on the *D*-scale, is 638.

A FINAL SUGGESTION At first glance, the operation of the slide rule and the instructions appearing here may appear confusing. Do not let that discourage you. Spend a few minutes each day with the slide rule, trying to master one operation at a time. Do not proceed to a new operation before you have completely understood the one preceding it. Using your own slide rule, follow the illustrative examples given with each operation step by step. This is the best way to learn, the easiest way to learn, the slide rule operations. There are thousands of applications of the use of the slide rule to common every-day problems. Use your slide rule at work, and in your home, and you will soon find it an indispensable tool for rapid calculating.

TEST YOUR ABILITY TO USE THE SLIDE RULE WITH THESE PROBLEMS

Editor's note: Do not attempt to work any of these problems until you have completely mastered the operations described on the preceding pages.

36 A gear 18″ in diameter has a circular pitch of $1\frac{1}{4}''$. How many teeth does the gear have on it?

$$\frac{3.14\times18}{1.25}=\text{Number of teeth on the gear.}$$

37 At what R.P.M. should a $1\frac{1}{4}''$ high speed drill be run to give a cutting speed on 80 ft. per minute?

$$\frac{80\times12}{3.1416\times1.25}=\text{R.P.M.}$$

38 How many feet per minute does a 22″ wheel cover when making 166 revolutions per minute?

$$\frac{3.14\times22\times166}{12}=\text{Feet per minute.}$$

39 A right triangle has sides 8.8 and 4.21 inches respectively. What is the length of the hypotenuse?

$$\sqrt{8.8^2\div4.21^2}=\text{Length of hypotenuse.}$$

40 A water tank is 15.6×8.6×3.3 feet. How many gallons of water will it hold? There are 7.48 gallons in a cubic foot.

41 Solve by means of the slide rule:

$$\sqrt{\frac{197.8\times14.675}{63.4}}.$$

The Measuring Rod

DECIMALS

1 Divide a 16-foot bar into 42 parts. Find the measurement of each part to the nearest hundredth of an inch.

2 A wire fence consists of 18 sections, each 9.25 feet long. How long is the fence?

3 A 15-story building is 148.55 feet high. What is the average height of each story? Give your result to one decimal place.

4 An iron bar weighs 1.965 lb. per ft. Determine the cost of a 15.75-ft. iron bar when the price per pound is $0.075 per pound.

5 The distance across the flats of a hexagonal-head machine bolt is 1.5 times the diameter of the bolt, plus 0.125″. Determine the distance across the flats of (a) a $\frac{1''}{4}$ bolt, (b) a $\frac{7''}{16}$ bolt, (c) a $\frac{5''}{8}$ bolt.

6 How many sheets of metal, each 0.0670 in. thick, are there in a pile 2 ft. 5.5 in. high?

7 A steel bridge is made up of nine sections, and the struts are equally spaced. The total length of the bridge is 272′. How long is each part to the nearest hundredth of a foot? (Fig. 9.)

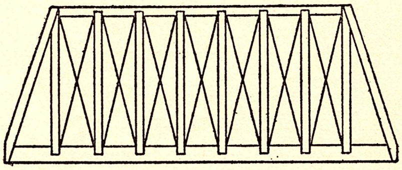


Fig. 9

8 A bronze alloy contains the following amounts of metals: copper 0.89 pound, tin 0.15, and manganese 0.015 pound. How much of each metal is there in a screw-propeller weighing 2459.4 pounds? Give the results to the nearest hundredth.

9 How many pounds of steel are there in a steel shaft $2\frac{3''}{4}$ in diameter and 10″ in length, if steel of this diameter weighs 13.6 lbs. per linear foot?

10 Turpentine weighs 0.0315 lbs. per cu. in. What is the weight of the turpentine that will fill a 50-gallon drum? (1 gallon = 231 cu. in.)

PERCENTAGES

11 There is a wastage of $2\frac{1}{4}\%$ in cutting iron bars. What is the total wastage in cutting 1000 bars, each 7 ft. 6 in. long?

12 A line shaft revolves at 224 r.p.m. It is found necessary to reduce this 25%. Determine the r.p.m. after the reduction is made.

13 The top speed of an aircraft at 7000 feet is 300 mph. At 11,000 feet, the top speed has increased 8%. What is the top speed at this altitude?

14 An airplane takes 94 minutes for a reconnaissance flight. If the speed

had been increased 15%, how long would the operation have lasted?

15 Water, in freezing, expands 9% of its volume. How many gallons of water are necessary to make 240 cubic feet of ice?

16 A gasoline engine is found to be only 68% efficient. If it is rated at 110 horsepower, what is the actual horsepower developed?

17 If at a given r.p.m. the propeller is 78% efficient and the motor develops 160 horsepower, how many horsepower are actually developed?

18 A certain alloy is composed of 75 parts of tin, 18.6 parts of zinc, and 2.4 parts of aluminum. What is the per cent of (a) zinc, (b) tin, (c) aluminum?

19 At 1,000 revolutions per minute, a propeller used 80% of the horsepower developed. If the engine develops 1,500 horsepower at 1,000 rpm, how many horsepower are used by the propeller?

20 An I-beam expands 0.01% of its length when exposed to the heat of the sun. Determine the increase in the length of an I-beam 26′7″ long.

21 Bronze used for bearings is composed of 82% copper, 2% zinc, and 16% tin. How many pounds of each of these metals are there in 24 bronze bars that weigh 94.75 pounds each?

22 A casting is machined, and in the process its weight is reduced by $7\frac{1}{2}\%$. If the finished casting weighs 348 pounds, find the weight of the original casting.

23 Out of 9600 small parts manufactured, $1\frac{1}{2}\%$ had to be discarded because of faulty work. How many usable parts resulted from the job?

24 If at a given r.p.m. the propeller is 78% efficient and the motor develops 165 HP, (a) how many HP are actually usable? (b) how many HP are lost?

25 The population of the United States in 1890 was about 63,000,000 people. This is what per cent of the present population (estimated at 140,000,000)?

26 At a pressure altitude of 20,000 ft. with the air temperature at 10° below zero, the calibrated air speed is 200 knots, and the true air speed is 282 knots. What is the per cent of increase in the air speeds?

27 The government orders an aircraft manufacturing company to modify the wing-span of a certain plane. If the original wing span is 53 feet 5 inches, and this is to be reduced $2\frac{1}{2}\%$, find the new wing-span.

ALIQUOTS

28 A defense worker works for 60 hours in one week at an average hourly wage of 75¢ an hour. How much does he receive at the end of the week?

29 At $\$1.12\frac{1}{2}$ a box, find the cost of 24 boxes of shell casings.

30 What is the cost of two dozen Civilian Defense Training Manuals at $33\frac{1}{3}$¢ each?

31 If 12 gas masks are worth $72.00, find the cost of (a) 6 gas masks; (b) 9 gas masks; (c) 4 gas masks; (d) 10 gas masks.

32 Find the cost of $7\frac{1}{2}$ yards of rubber tubing at $66\frac{2}{3}$¢ a yard.

33 If $87\frac{1}{2}\%$ is a fair average for production of rivets, how many rivets would have to be rejected in a total production of 72,000?

34 At $66\frac{2}{3}$ cents an hour, how much would a laborer receive for 42 hours' work?

35 Allowing $12\frac{1}{2}\%$ for shrinkage, how much material would have to be purchased to provide 16 lengths of 2 feet each?

AVERAGES

36 Find the average of the following weights: 67 pounds, 88 pounds, 76 pounds, 55 pounds, 59 pounds.

37 In an Army Air Corps examination, the students made the following grades: 77%, 61%, 92%, 88%, 68%, 45%, 73%, 84%. What was the average grade?

38 In two successive flights a Curtiss-Wright bomber covers 565 miles at 210 m.p.h., and 770 miles at 215 m.p.h. Determine its average speed for the two journeys.

39 The thermometer readings at noon for a certain week are 75 degrees; 77 degrees; 79 degrees; 80 degrees; 81 degrees; 76 degrees; 72 degrees. What is the average temperature at noon for the week?

40 Determine the average output per hour if the screw machines in a shop finish bolts as follows: 370, 384, 277, 431, 437.

41 In the rainfall report for New York City the Weather Bureau reports that during the month of June it rained on five different days, as follows: 0.25 in., 0.365 in., 1.5 in., 0.116 in., and 0.05 in. Find the average rainfall for that month.

42 Three measurements were made of a wrist pin, with the readings as follows: 0.733″, 0.750″, 0.749″. What is the correct diameter of the wrist pin?

43 Of the following dimensions, which is nearest the average: 0.315″, 0.353″, 0.337″, 0.328″, 0.346″?

44 A Navy bomber covers 292 miles in the first hour of its flight, 508 miles in the next $2\frac{1}{2}$ hours of flight, and 378 miles in the final $1\frac{1}{2}$ hours of flight. Find its average speed for the entire operation.

DENOMINATE NUMBERS

45 Each of the sides of a square field is 160 rods long. How many square yards does it contain?

46 What is the volume of a rectangular bin 6 feet high, 11 feet long, and 18 feet wide?

47 Divide a sheet of steel 16 ft. long into eleven equal parts. Make no

allowance for wastage in cutting. Give the answer to the nearest thirty-second of an inch.

48 An Army officer sent a letter by air mail to Second Corps Area Headquarters in New York from his station at El Paso, Texas. The letter left the Texas airport at 1:00 p.m. and arrived in New York the following day at 2:40 a.m. How long did it take the letter to travel from Texas to New York?

49 A storage closet on a battleship is 12 ft. × 14 ft. × 10 ft. How many cubic yards does it contain?

50 A war production plant allowed its employees to work 40 hours a week. If an employee worked a total of 28 hours, 10 minutes from Monday through Wednesday of a particular week, what would be the time that he could work on Thursday and Friday of that week, considering that he put in no overtime?

51 How many square feet of plastering will be required for the four walls and ceiling of an inside room 25 feet long, 20 feet wide, and 11 feet high?

52 A locker room in a submarine is 12′6″ wide. There are to be installed in this space 13 lockers of equal width. Determine the width of each to the nearest sixteenth of an inch.

53 If the height of the ground floor of a 10-story building is 15 ft., 8 in. and each of the other stories is 10 ft., $8\frac{1}{2}$ in., how high is the building?

RATIO AND PROPORTION

54 If it takes 12 days for 4 soldiers to build a camp arsenal, in what time can 9 soldiers do it?

55 A photographer has enlarged a 5 in. by 12 in. picture so that its width is 20 in. What is the length?

56 A tree casts a shadow of 40 ft. A post that is 4 ft. high casts a shadow of 7 ft. How high is the tree?

57 When a pole 32 ft. high casts a shadow 14 ft. long, how long is the shadow cast by a nearby tree which is 64 ft. high?

58 At a Navy benefit receipts were $552 and the expenses were twice as great as the profits. What were the profits and the expenses?

59 One man can do a piece of work in 5 days and another can do the same work in 3 days. If the men are paid in proportion to the work they do and if the first man receives $5.22 a day, what does the second man receive?

60 A bar 4 feet 3 inches long is to be cut into three parts in the ratio 4:3:2. What are the lengths of the parts?

61 An airplane flying 120 m.p.h. covers a distance in 3 hr. 15 min. At what rate would it have to fly to cover the same distance in 2 hr. 30 min.?

62 If $\frac{3}{4}$ of an inch on a map represents 28 mi., what distance does $4\frac{1}{2}$ in. represent on the map?

63 If 75 feet of wire weigh 2.75 pounds, how many feet are there in a coil of the same kind of wire weighing 13.85 pounds?

64 If a beam 4″ thick and carrying a safe load of 3500 pounds is to be replaced by another beam of identical material, having the same length and width, to carry a safe load of 7500 pounds, what should be the thickness of the new beam?

Odd Problems For Off Hours

Not Acceptable in a V-letter

15 Private Bradwin found his first weeks at camp rather difficult because he couldn't readjust his spending to his new rate of pay. In desperation, he wrote home:

```
  S E N D
  M O R E
M O N E Y
```

If each letter represents a figure, the same letter representing the same figure each time, how much money did he request his father to send?

As the Romans Do

16 Under what conditions are these statements true? (a) Two-thirds of six is nine. (b) One-half of five is four. (c) Seven is one-half of twelve. (d) One-half of eleven is six.

A Heap of Exercise

17 Take 10 pennies (or poker chips or other disks). Lay them in a straight row. Ask your friends to stack them in piles of 2 each, each piece to be jumped over 2 other pieces. This one ought to be good for an hour's pastime.

Cutting Costs

18 Corporal Midgeway needed a shave and a haircut. He began to shop around to learn where he could get the best prices. He found that Pierre Couteau's barber shop made the following charges: haircut, 40¢; shave, 20¢. Across the street at Nick Panagra's, the prices were: haircut, 35¢; shave, 25¢. He discovered that, by getting a haircut at Nick's and a shave at Pierre's, he could reduce his total expenditure. Accordingly, he went to Nick's and paid 35¢ for a haircut, thereby saving a nickel on the cost of the haircut, and then went to Pierre's and paid 20¢ for a shave, thus saving another nickel on the cost of the shave. In spite of the fact that he thus thought he had saved 10¢, he discovered that his total cost was only 5¢ less than the total cost would have been at either barber shop. What happened to the other nickel?

Nine to One Hundred

19 Arrange the nine digits so that they add up to 100. (Two solutions without fractions; almost limitless possibilities if fractions are employed.)

Five plus Six equals Nine

20 Add five to six in such a way that the number resulting is nine.

A Short Race, but a Fast One

21 Two coast guards decided to race. Bill, being faster, gave Andy a handicap of 27 steps from his own starting-point. Andy took 8 steps to Bill's 5, but 2 of Bill's steps were equal in distance to 5 of Andy's. The race ended in a tie. How many steps did Bill take from start to finish?

Not a Third-rate Problem

22 Write 24 with three equal figures, none of them being 8.

One Way to Save Shoe Leather

23 The story is told of an old gentleman who used to take a daily constitutional around the block. So regular was he in his habits that the neighbors took to setting their clocks by him. One day, he failed to pass a certain house. Next day, the housewife, seeing him go by, called to ask him if he had been ill the previous day.

"Oh, no", was his reply, "but, you see, it's this way: I'm getting too feeble to walk the whole way around the block now, so I walk half way around and then go home again."

How much distance did he actually save?

This One Can Be Done!

24 How can you give 35 cents in change with two coins, one of which is not a 10-cent piece?

The Solutions to these puzzles will appear in Issue Number Three

ANSWERS TO PUZZLE-PROBLEMS IN ISSUE NUMBER ONE

1 This is done by a trick of manipulation. The sum of the digits from 1 to 9, inclusive, is 45. Set down the digits from 9 to 1, in reverse order, and from that number subtract the digits from 1 to 9, in order. The answer will be found also to contain the nine digits, although in a different order.

$$\begin{array}{r} 987654321 \\ -123456789 \\ \hline 864197532 \end{array}$$

2 The sailor, having \$45 to begin with, paid \$1 to get in at the first door, then having \$44 left. He paid \$22 at the canteen and \$1 to get out, retaining \$21. He paid \$1 at the second door, \$10 inside, and \$1 to get out, having \$9 left. Paying \$1 at the third door, he spent \$4 inside, and \$1 to get out. Of the remaining \$3, he paid \$1 to get in, \$1 for purchases, and \$1 to get out.

4 His profits amount to \$60. Ending with \$330, he was \$60 better off than at the conclusion of his first sale.

5 Three cats! Did you miss on this one?

6 The least common multiple of 2, 3, 4, 5, and 6 is 60. The lowest multiple of this which could be used in this problem is 300, which, with 1 added, gives 301.

7 Designate the Americans as A_1, A_2, and A_3 and the Japs as J_1, J_2, and J_3. The trips would be made as follows:

Over	*Left Across*	*Return*
A_1, J_1	J_1	A_1
A_2, J_2	A_2, J_2	J_1
A_3, J_3	A_3	J_3
J_1, J_3	J_1, J_3	J_2
A_1, J_2	(All over)	

8 Six trips. (Two of the ears which the squirrel carries in are on his head!)

9 By adding one of his own horses to the number in the stable, the lawyer was able to perform the division, giving 9, 6, and 2 horses to the sons, and then driving his own horse home again.

10 $\left[\frac{1}{3}+\left(\frac{1}{2}\times\frac{1}{3}\right)\right]\frac{1}{2}\times 10=\frac{1}{2}\times\frac{1}{2}\times 10=2\frac{1}{2}.$

11 With 783 eggs in her basket, the old woman sold successively $\frac{783}{2}+\frac{1}{2}=392$, $\frac{391}{2}+\frac{1}{2}=196$, $\frac{195}{2}+\frac{1}{2}=98$, and $\frac{97}{2}+\frac{1}{2}=49$. $(392+196+98+49+48=783.)$

12 The grocer lost \$45 plus the cost of the groceries.

13 $\frac{3}{16}$.

14 (a) $99\frac{9}{9}$; (b) $4\times 1\frac{1}{1}=4\frac{4}{4}=4+1=5$.

Strange Ways With Numbers

The Famous Fifteen Puzzle

6 For half a century or more, men and women all over the world have found entertainment in working out an extremely interesting and sometimes baffling puzzle. It consists of a square shallow box containing fifteen blocks, and divided into sixteen partitions, so that only one block may be moved at a time. The blocks, numbered from 1 to 15, are placed in the box at random as shown in the figure below. The object of the puzzle is to arrange the blocks in regular order, so that the top row of blocks will read 1, 2, 3, 4; the second row will read 5, 6, 7, 8; the third row will read 9, 10, 11, 12; and the fourth row will read 13, 14, 15, the space for the 16th block being empty.

1	3	2	4
6	5	8	7
10	12	9	11
13	14	15	

Fig. 10

It is a great deal of fun to shift the blocks until block number 1 is brought to the upper left-hand corner, then to bring block number 2 next to it, and so on until the problem is solved. It sometimes happens, however, that, when the lowest row of blocks is reached, the block number 15 is where block number 14 should be, and block number 14 is where block number 15 should be. When such a result is reached, it is impossible to solve the problem, since there is no way to switch the positions of the two blocks without destroying the other sequences.

This puzzle comes under that branch of mathematics known as permutations and combinations. The number of ways in which the fifteen blocks can be put at random in the box has been determined to be 1,307,674,368,121. A mathematical analysis of the puzzle has shown us that only one-half of these groupings of the blocks will permit the blocks to be moved so that the correct solution may be reached.

The famous Fifteen Puzzle is sold by most novelty shops for a small price. However, it is of such a simple design that it can be easily constructed with materials available at home. Beginning with the blocks in the position shown in the figure, try to solve this puzzle yourself. The correct solution, with all of the steps necessary to achieve it, will appear in an early issue. Do not try to solve this puzzle in a few minutes; it is difficult—but it can be done.

The Mystic Eight

7 In the previous issue, we showed you one of the strangest mathematical curiosities, the case of the mystic one. Here is another curiosity which is almost as baffling as the mystic one, and should prove interesting, the mystic eight:

$$9\times9+7=88$$
$$9\times98+6=888$$
$$9\times987+5=8888$$
$$9\times9876+4=88888$$
$$9\times98765+3=888888$$
$$9\times987654+2=8888888$$
$$9\times9876543+1=88888888$$
$$9\times98765432+0=888888888$$

Note this peculiarity: In each instance, the number of the digits in the solution is one more than the number of digits in the number multiplied by 9.

We Never Work At All!

8 Like words, figures tell many an interesting story. Here, for example, is an interesting curiosity which proves that you never work at all:

In a year there are............	365 days
If you relax 8 hours a day, that equals..................	122 days
Leaving....................	243 days
If you sleep 8 hours a day, that equals..................	122 days
Leaving....................	121 days
In a year there are 52 Sundays....................	52 days
Leaving....................	69 days
You take off a half-day on Saturdays, totaling...........	26 days
Leaving....................	43 days
You take $1\frac{1}{2}$ hours a day for meals, totaling...............	28 days
Leaving....................	15 days
You take two weeks' vacation every year, totaling...........	14 days
Leaving....................	1 day
This remaining day is the first Monday in September (Labor Day)......................	1 day
Leaving....................	0 days

—and so there is no time at all left for work, all of your time being devoted to idle pleasures!

Take a Number from One to Nine

9 Here is an interesting stunt with numbers that you can use to mystify your friends. This is how it works: you ask someone to select any number between 1 and 9; when they have selected this number, you tell them to perform a certain multiplication which will give them a product consisting of the selected number repeated nine times.

As a case in point, suppose your friend selects the number 7, you instruct him to multiply 12,345,679 by the magic multiplier, 63.

$$\begin{array}{r} 12{,}345{,}679 \\ \times 63 \\ \hline 37037037 \\ 74074074 \\ \hline 777{,}777{,}777 \end{array}$$

Notice that in this multiplication there is an additional peculiarity: the figures, 370 and 740, are repeated in the process of making the multiplication. This curious repetition does not appear in all cases. Note also that in the multiplicand the figure, 8, does not appear.

This same stunt with numbers can be performed to produce any other figure between 1 and 9. In the following chart, you can find the magic multipliers which must be used to perform this trick. You do not need to memorize these numbers, for, on inspection, you will note that they are all multiples of 9. Note that each of the magic multipliers is simply the number selected multiplied by 9. As an example, if the number 1 is selected, the magic multiplier is 1×9, or 9; if the number 3 is selected, the magic multiplier is 3×9, or 27, and so on.

Magic Multiplier Solution

$$9\times12{,}345{,}679=111{,}111{,}111$$
$$18\times12{,}345{,}679=222{,}222{,}222$$
$$27\times12{,}345{,}679=333{,}333{,}333$$
$$36\times12{,}345{,}679=444{,}444{,}444$$
$$45\times12{,}345{,}679=555{,}555{,}555$$
$$54\times12{,}345{,}679=666{,}666{,}666$$
$$63\times12{,}345{,}679=777{,}777{,}777$$
$$72\times12{,}345{,}679=888{,}888{,}888$$
$$81\times12{,}345{,}679=999{,}999{,}999$$

Solutions to Questions and Exercises

BASIC ARITHMETIC

ADDITION

1 14
2 11
3 19
4 17
5 17
6 15
7 15
8 24
9 18
10 21
11 19
12 20
13 22
14 21
15 21
16 26
18 99
19 126
20 169
21 187
22 218
23 202
24 886
25 2,302
26 2,171
27 2,057
28 2,982
29 2,480
30 17,904
31 24,609
32 41,524
33 43,716
34 29,483
35 37,321
36 33,155
37 42
38 135
39 191
40 288
41 660
42 599,647
43 1,019,766

SUBTRACTION

47 8,215
48 3,319
49 7,102
50 6,033
51 1,003
52 1,304
53 8,013
54 17,893
55 27,767
56 297
57 340
58 2,874,977
59 2,425
60 14,137
61 5,251

MULTIPLICATION

65 189
66 288
67 728
68 72
69 286
70 1,482
71 3,486
72 2,620
73 66,304
74 317,952
75 634,766
76 205,590
77 455,972
78 2,428,677
79 660,625
80 1,395,968
81 537,299,646
82 138,701,983
83 797,517,495
84 890,354,862
85 966
86 910
87 2,400
88 405
89 128
90 $621.00
91 42,000

SHORT DIVISION

95 231
96 212
97 313
98 318
99 106
100 54
101 81
102 85
103 42
104 123
105 134
106 32
107 1,464
108 855
109 7789
110 3,812
111 12,053
112 9,301
113 1,262
114 5,215

LONG DIVISION

115 21
116 203
117 203
118 $11\frac{46}{68}$
119 $160\frac{3}{13}$
120 $72\frac{58}{78}$
121 $44\frac{58}{136}$
122 $129\frac{66}{545}$
123 $110\frac{345}{619}$
124 $296\frac{184}{208}$
125 $116\frac{354}{428}$
126 $13\frac{381}{961}$
127 4
128 11
129 30
130 4,2,7
131 8
132 44
133 9

ARITHMETIC FUNDAMENTALS

136 7
137 9
138 212 cattle
394 sheep
40 lambs
139 35
140 6,757
141 236,871
142 $126
143 10 hrs.
20 min.
144 1141°
145 9 hrs.
146 18
147 20
148 45 lbs.
149 $332
150 4,185
151 27
152 $81.60
153 74,784
154 57,150,000
155 187,928 lbs. per sq. in.
156 $36,977,680,088

FRACTIONS

MULTIPLICATION OF FRACTIONS

Exercises

1 $\frac{4}{5}$
2 $1\frac{2}{3}$
3 3
4 $\frac{1}{2}$
5 $\frac{3}{5}$
6 $1\frac{2}{3}$
7 $\frac{2}{3}$
8 $3\frac{1}{3}$
9 $\frac{5}{6}$
10 $1\frac{1}{5}$
11 2
12 $\frac{1}{2}$
13 $\frac{3}{5}$
14 $1\frac{1}{5}$
15 $\frac{2}{3}$
16 $3\frac{1}{3}$
17 $\frac{1}{15}$
18 $\frac{4}{15}$
19 $\frac{1}{2}$
20 $\frac{3}{7}$
21 $\frac{1}{10}$
22 $\frac{2}{7}$
23 $\frac{3}{8}$
24 $\frac{9}{25}$
25 $\frac{2}{15}$
26 $\frac{1}{3}$
27 $1\frac{1}{8}$
28 $1\frac{1}{2}$
29 $\frac{5}{7}$
30 $2\frac{3}{5}$
31 $\frac{7}{10}$
32 $1\frac{1}{10}$
33 3
34 $\frac{2}{3}$
35 $1\frac{1}{15}$
36 $3\frac{1}{2}$
37 $\frac{5}{8}$
38 $\frac{81}{100}$
39 $1\frac{1}{2}$
40 2
41 $\frac{7}{10}$
42 $1\frac{1}{6}$
43 $\frac{1}{2}$
44 $\frac{2}{3}$
45 $\frac{3}{4}$
46 $\frac{9}{10}$
47 $1\frac{1}{2}$
48 2
49 $\frac{3}{4}$
50 $\frac{2}{3}$
51 $\frac{8}{9}$
52 $1\frac{11}{12}$
53 $\frac{2}{3}$
54 $\frac{3}{4}$
55 $1\frac{1}{2}$
56 $4\frac{4}{5}$
57 4
58 $3\frac{1}{2}$
59 $10\frac{1}{2}$
60 $5\frac{1}{3}$
61 4
62 $3\frac{1}{2}$
63 $11\frac{1}{4}$
64 $1\frac{19}{30}$
65 $3\frac{1}{2}$
66 $2\frac{13}{16}$
67 $1\frac{3}{5}$
68 $1\frac{1}{3}$
69 $3\frac{1}{9}$
70 3

Problems

71 135 **72** 16 **73** 27 **74** $2\frac{1}{6}$ **75** 75

DIVISION OF FRACTIONS

Exercises

76 $\frac{1}{6}$
77 $\frac{2}{35}$
78 $\frac{1}{5}$
79 $\frac{2}{7}$
80 $\frac{1}{15}$
81 6
82 $4\frac{1}{2}$
83 25
84 $5\frac{1}{2}$
85 $10\frac{2}{3}$
86 $\frac{2}{3}$
87 $1\frac{1}{2}$
88 3
89 $\frac{1}{3}$
90 $\frac{5}{6}$
91 $1\frac{2}{3}$
92 $\frac{2}{3}$
93 $1\frac{1}{2}$
94 $\frac{3}{4}$
95 $\frac{9}{20}$
96 $\frac{1}{2}$
97 3
98 $\frac{1}{3}$
99 $\frac{2}{9}$
100 $\frac{9}{10}$
101 2
102 $\frac{1}{2}$
103 $\frac{1}{2}$
104 $1\frac{1}{3}$
105 $1\frac{1}{24}$
106 $\frac{24}{25}$
107 $1\frac{1}{4}$
108 $\frac{4}{5}$
109 $\frac{3}{20}$
110 $\frac{2}{25}$
111 $\frac{5}{18}$
112 $\frac{9}{20}$
113 $\frac{1}{3}$
114 $\frac{50}{147}$
115 $\frac{5}{48}$
116 $1\frac{5}{13}$
117 $2\frac{4}{7}$
118 7
119 $3\frac{1}{2}$
120 $10\frac{1}{2}$
121 $1\frac{3}{4}$
122 $2\frac{4}{5}$
123 $2\frac{16}{27}$
124 $1\frac{8}{27}$
125 $1\frac{1}{3}$
126 $4\frac{2}{15}$
127 $5\frac{1}{3}$
128 $\frac{2}{3}$
129 $\frac{1}{2}$
130 $2\frac{2}{5}$
131 $\frac{4}{5}$

DIVISION OF FRACTIONS (continued)

Exercises (Continued)

132 $1\frac{1}{3}$
133 $\frac{3}{10}$
134 $1\frac{1}{14}$
135 $1\frac{1}{3}$
136 $4\frac{1}{2}$
137 2
138 $1\frac{1}{4}$
139 $1\frac{2}{3}$
140 $\frac{15}{17}$
141 $2\frac{1}{2}$
142 $\frac{8}{9}$
143 $1\frac{1}{8}$
144 $\frac{5}{7}$
145 $\frac{7}{8}$
146 $1\frac{1}{7}$
147 $1\frac{1}{24}$
148 2
149 $\frac{1}{2}$
150 $\frac{10}{17}$
151 $1\frac{7}{10}$
152 $\frac{39}{40}$
153 $1\frac{1}{39}$
154 $\frac{1}{2}$
155 2

Problems

156 $3\frac{3}{4}$ **157** 816 **158** 35 in. **159** $5\frac{5}{6}$ days

ADDITION OF FRACTIONS

Exercises

160 $\frac{2}{5}$
161 $\frac{1}{3}$
162 $1\frac{1}{5}$
163 $1\frac{2}{3}$
164 $1\frac{2}{5}$
165 $1\frac{1}{2}$
166 $2\frac{1}{3}$
167 2
168 $2\frac{2}{3}$
169 $3\frac{2}{5}$
170 $3\frac{1}{3}$
171 $4\frac{1}{5}$
172 4
173 $3\frac{3}{5}$
174 $1\frac{1}{2}$
175 $7\frac{1}{2}$
176 $\frac{3}{10}$
177 $\frac{2}{3}$
178 $1\frac{1}{2}$
179 $1\frac{1}{2}$
180 $1\frac{3}{4}$
181 $1\frac{1}{3}$
182 $2\frac{1}{4}$
183 $3\frac{1}{3}$
184 $2\frac{3}{5}$
185 $2\frac{14}{15}$
186 $4\frac{1}{4}$
187 $2\frac{5}{9}$
188 $\frac{5}{6}$
189 $1\frac{1}{10}$
190 $1\frac{9}{14}$
191 $2\frac{7}{10}$
192 $3\frac{5}{6}$
193 $4\frac{1}{6}$
194 $\frac{7}{12}$
195 $\frac{9}{40}$
196 $\frac{11}{18}$
197 $1\frac{1}{40}$
198 $1\frac{7}{12}$
199 $1\frac{9}{40}$
200 $1\frac{13}{36}$
201 $1\frac{29}{40}$
202 $3\frac{5}{12}$
203 $2\frac{7}{20}$
204 $4\frac{1}{12}$
205 $2\frac{1}{40}$

Problems

206 $1598\frac{3}{4}$ **207** $12\frac{1}{4}''$ **208** $1\frac{17}{24}''$ **209** $31\frac{3}{8}''$
210 $15\frac{1}{4}$

SUBTRACTION OF FRACTIONS

Exercises

211 $\frac{1}{5}$
212 $\frac{2}{3}$
213 0
214 $3\frac{1}{7}$
215 $7\frac{1}{2}$
216 5
217 $\frac{1}{3}$
218 $\frac{1}{5}$
219 $2\frac{2}{5}$
220 $2\frac{1}{2}$
221 1
222 $\frac{1}{2}$
223 $\frac{1}{3}$
224 $3\frac{2}{5}$
225 $5\frac{1}{2}$
226 $\frac{4}{5}$
227 $1\frac{1}{3}$
228 $4\frac{2}{3}$
229 $1\frac{2}{3}$
230 $\frac{1}{2}$
231 $3\frac{1}{4}$
232 $\frac{1}{2}$
233 $6\frac{1}{2}$
234 $1\frac{4}{9}$
235 $1\frac{1}{2}$
236 $\frac{1}{6}$
237 $\frac{1}{4}$
238 $2\frac{1}{4}$
239 $5\frac{1}{2}$
240 $\frac{5}{12}$
241 $\frac{2}{5}$
242 $2\frac{1}{4}$
243 $4\frac{1}{2}$
244 $\frac{1}{2}$
245 $1\frac{4}{5}$
246 $8\frac{5}{8}$
247 $4\frac{1}{2}$
248 $\frac{5}{8}$
249 $\frac{2}{3}$
250 $2\frac{3}{4}$
251 $1\frac{1}{2}$
252 $\frac{4}{15}$
253 $2\frac{13}{14}$
254 $\frac{1}{6}$
255 $2\frac{4}{35}$
256 $2\frac{5}{6}$
257 $6\frac{1}{10}$
258 $\frac{7}{12}$
259 $1\frac{8}{15}$
260 $\frac{1}{12}$
261 $\frac{1}{40}$
262 $1\frac{2}{15}$
263 $4\frac{1}{15}$
264 $\frac{1}{15}$
265 $\frac{2}{15}$
266 $1\frac{1}{12}$
267 $1\frac{1}{15}$
268 $\frac{8}{15}$
269 $\frac{7}{15}$
270 $1\frac{11}{15}$
271 $1\frac{33}{40}$
272 $\frac{11}{15}$
273 $\frac{33}{40}$
274 $1\frac{11}{15}$

Problems

275 $\frac{7}{16}$ **276** $18\frac{19}{32}''$ **277** $\frac{13}{16}$ **278** $2\frac{9}{16}$

PROBLEMS ON FRACTIONS AND MIXED NUMBERS

279 $\frac{3}{4}$
280 $12\frac{53}{64}$
281 27 days
282 $2\frac{9}{16}$
283 222
284 928
285 94
286 2,400
287 $333\frac{1}{3}$
288 $4\frac{1}{2}$
289 $7\frac{1}{5}$ days
290 $\frac{11}{16}$
291 $59{,}572\frac{39}{64}$ lbs.
292 286.34 copper
47.72 tin
11.93 zinc
293 732 miles
294 $7\frac{21}{32}''$
295 (a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{5}{9}$
296 $68\frac{17}{40}$
297 $6\frac{15}{16}'$
298 $376\frac{1}{2}'$
299 80
300 $59\frac{5}{7}'$

NUMBERS THROUGH THE AGES

1 8 **3** 35 **5** 94 **7** 1,920 **9** 1,396
2 14 **4** 75 **6** 800 **8** 5,000 **10** 1,589

11 one-tenth
12 sixty-five hundredths
13 four hundred eighty-three thousandths
14 one thousand four hundred twenty-five ten-thousandths
15 one and eight hundred twelve thousandths
16 three and one thousand four hundred sixteen ten-thousandths
17 two and one thousandth
18 ten and four hundred fifty-six thousandths
19 one thousand one and one ten-thousandth
20 one hundred thousand and seven thousand six hundred forty-three ten-thousandths

THE MEASURING ROD

INTEGERS

1 59
2 40
3 39
4 3,493
5 3,589
6 7,189
7 3,457
8 26,209
9 115,707
10 17,297
11 2,781
12 4,797
13 1,207
14 5,692
15 3,077
16 40,091
17 4,117
18 324
19 134,784
20 323,541
21 7,268,142
22 5,091,072
23 2,108,925
24 4,823,937
25 $2{,}617\frac{27}{29}$
26 $1{,}059\frac{19}{21}$
27 $2{,}148\frac{3}{38}$
28 $76\frac{268}{297}$
29 $385\frac{11}{13}$

FRACTIONS AND MIXED NUMBERS

30 $2\frac{3}{4}$
31 $4\frac{1}{8}$
32 $2\frac{2}{3}$
33 $286\frac{5}{24}$
34 $232\frac{2}{9}$
35 $137\frac{11}{48}$
36 $206\frac{23}{96}$
37 $210\frac{21}{36}$
38 $255\frac{1}{16}$
39 $\frac{2}{3}$
40 $\frac{9}{16}$
41 $\frac{1}{6}$
42 $\frac{1}{6}$
43 $\frac{1}{2}$
44 $\frac{1}{2}$
45 $2\frac{1}{4}$
46 $7\frac{7}{40}$
47 $12\frac{3}{4}$
48 $11\frac{7}{8}$
49 $16\frac{5}{8}$
50 $16\frac{3}{8}$
51 $16\frac{1}{2}$
52 $8\frac{7}{16}$
53 $\frac{1}{20}$
54 $416\frac{2}{3}$
55 $1\frac{1}{4}$
56 $1\frac{3}{4}$
57 8
58 2
59 4
60 6
61 5
62 $1\frac{53}{124}$

Tables and Formulas

TABLE IX
DECIMAL EQUIVALENTS OF COMMON FRACTIONS

Fractions	Decimal	Fractions	Decimal	Fractions	Decimal
$\frac{1}{64}$	0.015625	$\frac{11}{32}$ $\frac{22}{64}$	0.34375	$\frac{43}{64}$	0.671875
$\frac{1}{32}$ $\frac{2}{64}$	0.03125	$\frac{23}{64}$	0.359375	$\frac{11}{16}$ $\frac{22}{32}$ $\frac{44}{64}$	0.6875
$\frac{3}{64}$	0.046875	$\frac{3}{8}$ $\frac{6}{16}$ $\frac{12}{32}$ $\frac{24}{64}$	0.375	$\frac{45}{64}$	0.703125
$\frac{1}{16}$ $\frac{2}{32}$ $\frac{4}{64}$	0.0625	$\frac{25}{64}$	0.390625	$\frac{23}{32}$ $\frac{46}{64}$	0.71875
$\frac{5}{64}$	0.078125	$\frac{13}{32}$ $\frac{26}{64}$	0.40625	$\frac{47}{64}$	0.734375
$\frac{3}{32}$ $\frac{6}{64}$	0.09375	$\frac{27}{64}$	0.421875	$\frac{3}{4}$ $\frac{6}{8}$ $\frac{12}{16}$ $\frac{24}{32}$ $\frac{48}{64}$	0.75
$\frac{7}{64}$	0.109375	$\frac{7}{16}$ $\frac{14}{32}$ $\frac{28}{64}$	0.4375	$\frac{49}{64}$	0.765625
$\frac{1}{8}$ $\frac{2}{16}$ $\frac{4}{32}$ $\frac{8}{64}$	0.125	$\frac{29}{64}$	0.453125	$\frac{25}{32}$ $\frac{50}{64}$	0.78125
$\frac{9}{64}$	0.140625	$\frac{15}{32}$ $\frac{30}{64}$	0.46875	$\frac{51}{64}$	0.796875
$\frac{5}{32}$ $\frac{10}{64}$	0.15625	$\frac{31}{64}$	0.484375	$\frac{13}{16}$ $\frac{26}{32}$ $\frac{52}{64}$	0.8125
$\frac{11}{64}$	0.171875	$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$ $\frac{8}{16}$ $\frac{16}{32}$ $\frac{32}{64}$	0.5	$\frac{53}{64}$	0.828125
$\frac{3}{16}$ $\frac{6}{32}$ $\frac{12}{64}$	0.1875	$\frac{33}{64}$	0.515625	$\frac{27}{32}$ $\frac{54}{64}$	0.84375
$\frac{13}{64}$	0.203125	$\frac{17}{32}$ $\frac{34}{64}$	0.53125	$\frac{55}{64}$	0.859375
$\frac{7}{32}$ $\frac{14}{64}$	0.21875	$\frac{35}{64}$	0.546875	$\frac{7}{8}$ $\frac{14}{16}$ $\frac{28}{32}$ $\frac{56}{64}$	0.875
$\frac{15}{64}$	0.234375	$\frac{9}{16}$ $\frac{18}{32}$ $\frac{36}{64}$	0.5625	$\frac{57}{64}$	0.890625
$\frac{1}{4}$ $\frac{2}{8}$ $\frac{4}{16}$ $\frac{8}{32}$ $\frac{16}{64}$	0.25	$\frac{37}{64}$	0.578125	$\frac{29}{32}$ $\frac{58}{64}$	0.90625
$\frac{17}{64}$	0.265625	$\frac{19}{32}$ $\frac{38}{64}$	0.59375	$\frac{59}{64}$	0.921875
$\frac{9}{32}$ $\frac{18}{64}$	0.28125	$\frac{39}{64}$	0.609375	$\frac{15}{16}$ $\frac{30}{32}$ $\frac{60}{64}$	0.9375
$\frac{19}{64}$	0.296875	$\frac{5}{8}$ $\frac{10}{16}$ $\frac{20}{32}$ $\frac{40}{64}$	0.625	$\frac{61}{64}$	0.953125
$\frac{5}{16}$ $\frac{10}{32}$ $\frac{20}{64}$	0.3125	$\frac{41}{64}$	0.640625	$\frac{31}{32}$ $\frac{62}{64}$	0.96875
$\frac{21}{64}$	0.328125	$\frac{21}{32}$ $\frac{42}{64}$	0.65625	$\frac{63}{64}$	0.984375

TABLE X
ALIQUOTS

Simple

$$2\tfrac{1}{2}\% = \frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$$

$$5\% = \frac{5}{100} = \frac{1}{20}$$

$$10\% = \frac{10}{100} = \frac{1}{10}$$

$$20\% = \frac{20}{100} = \frac{1}{5}$$

$$4\% = \frac{4}{100} = \frac{1}{25}$$

$$6\tfrac{1}{4}\% = \frac{6\frac{1}{4}}{100} = \frac{25}{400} = \frac{1}{16}$$

$$12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$$

$$25\% = \frac{25}{100} = \frac{1}{4}$$

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$8\tfrac{1}{3}\% = \frac{8\frac{1}{3}}{100} = \frac{1}{12}$$

$$16\tfrac{2}{3}\% = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$$

$$33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$$

Complex

$$30\% = \frac{30}{100} = \frac{3}{10}$$

$$40\% = \frac{40}{100} = \frac{2}{5}$$

$$60\% = \frac{60}{100} = \frac{3}{5}$$

$$70\% = \frac{70}{100} = \frac{7}{10}$$

$$80\% = \frac{80}{100} = \frac{4}{5}$$

$$90\% = \frac{90}{100} = \frac{9}{10}$$

$$37\tfrac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}$$

$$66\tfrac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$$

$$75\% = \frac{75}{100} = \frac{3}{4}$$

$$87\tfrac{1}{2}\% = \frac{87\frac{1}{2}}{100} = \frac{175}{200} = \frac{7}{8}$$

$$66\tfrac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{2}{3}$$

$$83\tfrac{1}{3}\% = \frac{83\frac{1}{3}}{100} = \frac{5}{6}$$

TABLE XI

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05691	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12711	13033	13354	13672	13988	14302
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21218	21484	21748	22011	22272	22531	22789
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646
19	27875	28103	28330	28556	28780	29003	29226	29447	28667	29885
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35794	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38738	38917	39094	39270	39445	39620
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42814	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44405	44560
28	44716	44871	45025	45179	45332	45485	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49416	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52505	52634	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54778	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57748	57864
38	57978	58093	58206	58320	58433	58546	58659	58771	58883	58995
39	59107	59219	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64247
44	64345	64444	64542	64640	64738	64836	64934	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65897	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69721	69810
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72510	72591	72673	72754	72835	72917	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957

TABLE XI (continued)

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74194	74273	74351	74429	74508	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75588	75664	75740	75816	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77306	77379	77452	77525	77597	77670	77743
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79519	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81225
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82608	82672	82737	82802	82866	82930	82995	83058	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84199	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84881	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024
76	88081	88138	88196	88253	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89210	89265	89321	89376	89432	89487	89542	89498	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91856
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94350	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95473	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	96000	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98901	98945	98990	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957

Glossary of Mathematical Terms

aliquot parts· exact divisors of a quantity. Many fractions may be treated as aliquot parts of 10, 100, or 1000, thereby shortening the process in which they are used. (See pages 74 and 125.)

antilogarithm: the number for which a logarithm stands. (See pages 90 and 96.)

average: the number obtained by dividing the sum of several quantities by the number of quantities. (See page 75.)

base a number which is raised to a given power The base of common logarithms is 10. (See page 90.)

characteristic: the integral part of a logarithm. (See page 91.)

cologarithm: the logarithm of the reciprocal of a number. It may be found by subtracting the logarithm of a number from 10. (See page 98.)

conversion: expressing a given quantity in terms of another quantity, (See page 65.)

cube of a number: the number multiplied by itself three times. (See pages 88, 100, and 111.) $2^3 = 2 \times 2 \times 2 = 8$.

cube root of a number: a number which, multiplied by itself three times, produces a given number. (See pages 100 and 112.)

denominate number: a number whose unit represents a unit of measure. (See pages 77 to 83.)

exponent: a number written above and to the right of another number or letter to indicate the number of times the expression is to be multiplied by itself. In the equation, $3^2 = 9$, the exponent is 2, indicating that 3 is to be multiplied by 3 to produce 9. (See page 88.)

extreme: the outside terms, or first and last terms of a proportion. (See page 84.)

hairline: a finely-etched line on the indicator of a slide rule, used as a guide in reading from the scale. (See page 101.)

indicator: the runner on the slide rule. (See page 101.)

inverse proportion: a statement of equality of two ratios such that the first fact concerning one object is to the second fact concerning the second object as the second fact concerning the first object is to the first fact concerning the second object. (See page 86.)

logarithms: a method of using exponents of a number (commonly 10) to assist in performing operations upon other numbers. (See page 90.)

mantissa: the decimal part of a logarithm. The mantissa is the only part of the logarithm usually given in "logarithmic tables". (See pages 92 and 103.)

metric: a system of measurement with meter as the basic unit.

negative powers: the power of the reciprocal of a number. $2^{-3} = \left(\frac{1}{2}\right)^3$. (See page 89.)

per cent: parts of one hundred. *Centum* is the Latin word for *one hundred.* Symbol: %. (See page 72.)

power: a number obtained by taking a given number a specified number of times as a factor. (See pages 88 and 99.)

proportion: a statement that two ratios are equal. (See page 84.)

proportional parts: fractional parts of the difference between two logarithms. (See page 95.)

radicand: the quantity under the radical sign. In the expression, $\sqrt{4}$, 4 is the radicand.

ratio: an expression of the relationship in size between two numbers or quantities. (See page 83.)

root: a number which, multiplied by itself a specified number of times, produces a given number. (See *cube root, square root.*)

runner: the movable indicator on the slide rule. (See page 101.)

scale: markings at regular and graduated intervals, to assist in reading lengths. (See page 101.)

slide rule: a ruler graduated in logarithmic scales, to assist in mechanical computation by means of logarithms. (See page 101.)

square: the result of multiplying a number by itself, as $2 \times 2 = 4$. (See pages 88, 99, and 108.)

square root: a number which, multiplied by itself' produces a given number; thus, $\sqrt{4} = 2$ since $2 \times 2 = 4$. (See pages 100 and 109.)

weighted average: an average obtained by taking into account certain facts about each of the quantities averaged. (See page 76.)

zero power: the result obtained by dividing a number by itself. The zero power, thus, is always equal to 1. (See page 89.)

See also pages 63 and 64 of Issue Number One.

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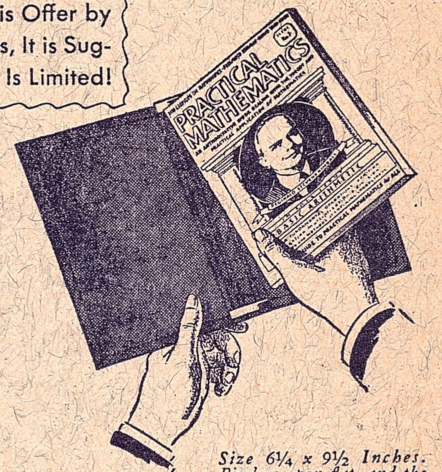

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